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Mathematical Models for Fertility and it's Application to the Egyption Data

By

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Abstract:

This paper investigates four mathematical models of fertility. The aim of this paper is to examine the suitability of these models to Egypt's data to deduce the suitable mathematical model of fertility in Egypt. Models are tested, a) The Gompertz model, b) the Hadwiger model, c)the truncated pearson type III curve (the incomplete gamma function) , and d) the pearsonian type I curve (the beta function). The data used to test the models are Egypt Demographic and Health Survey(EDHS), 1992, 1995, and 2000. The results indicate that the the truncated pearson type III curve is useful in describing fertility patterns in Egypt but the other models are not useful in describing fertility patterns in Egypt.

1- Introduction:

A study of fertility patterns has revealed the possibility of using many mathematical functions as a graduation of fertility distributions, such as the incomplete Gamma function (Luther, 1982; Nurul-Islam and Mallick, 1987), the Hadwiger function (Hoem et al, 1981; Chandola, Coleman, and Hiorns, 1999; Gage, 2000), the Gompertz function (Murphy and Nagnur, 1972; Faried, 1973; Pollard and Volkovics, 1992), and the Beta function (Mitra, 1967; Romanuik, 1973). This paper is interested in studying the all pervious models and it's application to Egypt's data.

In the second section, the Gompertz function , the Hadwiger function, the truncated pearson type III curve (the incomplete gamma function) , and the pearsonian type I curve (the beta function) briefly presented. In the third section, the methods will be used for estimating the parameters of each model are presented. In the fourth section, presents the results of the application of models on Egypt's data (EDHS, 1992, 1995, and 2000). Section five introduces the conclusion and recommendation .

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2- **The Models:**

(i) The Hadwiger function is an adaptation of the inverse Gaussian or Wald density. It is

$$H(X)=C\left\{\frac{b_1}{b_2\sqrt{\pi}}\left(\frac{b_2}{x-b_3}\right)^{3/2}\exp\left(-b_1^2\left(\frac{-b_2}{x-b_3}+\frac{x-b_3}{b_2}-2\right)\right)\right\}$$

For $X > b_3$,

where:

C is the Total fertility rate (TFR), (i.e. The sum of ASFRby five year age groups)

X is the age of mothers (i.e. x = 15, 16, ..., 49),

b_1, b_2, and b_3 are the parameters of the model but have no particular demographic interpretation, and

H(x) is the age-specific fertility rate at any age x (ASFR).

(ii) The Gompertz function is

$$Y(X) = KA^{B^{(X-X_0)}}$$

where: the origin is at X_o

Y(x) is the cumulative fertility rate,

X is the age of mothers (i.e. t = 15, 16, ..., 49),

K, A, B are the parameters of the model.

Murphy and Nagnur (1972) provided a demographic interpretation of the three parameters of Gompertz function as

K: total fertility rate,

A: the proportion of total fertility completed at the origin X_o,

B: an indication of the variance of the distribution (the spread) of fertility over the age span.

When the Gompertz function is interpreted as the cumulative fertility rate, the first derivative with respect to age represents the age specific fertility rate (ASFR).

$$ASFR = \frac{dY(x)}{dx} = KA^{B^{(x-xo)}}Ln\left(A^{B^{(x-xo)}}\right)Ln(B).$$

(iii) The truncated pearson type III curve

$$f(x) = \begin{cases} K(x-s)^2 e^{-(x-s)/m} & s \leq x \leq u \\ 0 & Otherwise \end{cases}$$

where:

K is the level of fertility (proportional to the TFR),

s is the initial age of fertility,

m is the distance between the initial and peak ages of fertility, and

u is the final age of fertility (The point of truncation).

(iv) The pearsonian type I curve

$$y = y_0(1+x/a_1)^{m1}(1-x/a_2)^{m2}$$

where: the origin is at mode; $-a_1 < x < a_2$

y_0 is the modal ordinate or is the modal fertility rate, which in human population falls between 20 and 30 years of age;

a_1, and a_2 jointly determine the reproductive interval; and

m_1 and m_2 determine the shape of the fertility curve

3- <u>The Method Used for Estimating The Parameters of Each Model</u>:

In this section, an attempt is made to introduce the methods used for estimating the parameters of Gompertz function, Hadwiger function, the pearson type III curve, and pearsonian type I curve. There are several methods used for estimating the parameters of the Gompertz function. The methods are a) the method of selected points (Murphy and Nagnur, 1972), b) the method of moments (Pollard and Valkovics (1992), and c) the iterative procedure (Murphy and Nagnur, 1972). Although the selected points method may provide good fit to the data, there always remains an element of uncertainty in such procedure (Mitra, 1967). Pollard and Valkovics (1992) stated that the moment method results in a poor fit,

because the Gompertz curve is simply the wrong shape, so used the iterative procedure (Murphy and Nagnur, 1972) for estimating the parameters of Gompertz function. We fixed K at the total fertility rate and iterate only on the other two constants (A, B). The parameters of

Hadwiger function are estimated by solving the three equations which represent the mean, the variance, and the mode of Hadwiger density function

$$\text{mean}=b_2+b_3$$

$$\text{variance} =\frac{1}{2}\, b_2^2 / b_1^2$$

$$\text{mode}= b_3+b_2\,\frac{(1+\frac{16}{9}b_1^4)^{1/2}-1}{\frac{4}{3}b_1^2}$$

Also the parameters of pearson type III curve(i.e. u, s, m, and k) are estimated as follow

$$\text{(i)}\ u = \begin{cases} 43 & if \quad r_1 > 2.515 \\ 46 & if \quad r_1 \le 2.515 \end{cases}$$

where r_1=f(35-39)/f(40-44), and f(35-39), and f(40-44) are the observed ASFRs at the age group 35-39 ,and 40-44 respectively.

(ii) The initial age of fertility s can be determine by several methods:

1. The method using MA (mean age of mother) and ratio of ASFR.
2. The method using MA and ratio of mean parity.
3. The method using two mean parities .

(iii)To determine m, the equation

$$MA = S + \frac{(u-s)^3 + 1.5(u-s)^2 m + 1.5(u-s)m^2 + 0.75m^3 - 0.75m^3 e^{2(u-s)/m}}{(u-s)^2 + (u-s)m + 0.5m^2 - 0.5m^2 e^{2(u-s)/m}}$$

can be solved by iterative method if MA is known, u and s are determined, to solve the pervious equation can be use max {MA- 0.852-5.75, 0.6667 (MA-s)} as starting value of m.

(iv) After determination of u, s and m, the parameter k which measuring the intensity of fertility can be determine by

$$k = \frac{4TFR}{m\{m^2 - 2(u-s)^2 + 0.5m^2)EXP(-2(u-s)/m)\}}$$

(more detials about the estimated parameters of truncated pearson type III curve see: Luther,1982). Finally, the parameters of pearsonian type I curve (i.e. a_1, a_2, m_1, and m_2) can be estimated as follow Mitra and Romaniuk (1973) stated that In Elderton (1930) procedure, the constants a_1, a_2, m_1, and m_2 are calculated from the first four moments of the frequency distribution, that is , from the mean, variance , skewness, and kurtosis, Mitra (1967) developed a procedure that reduces the required number of moments to the first two, but this procedure assumes a fixed age interval of fertility, later, Mitra and Romaniuis (1973) secceded in deriving the constants from only the mean and modal age of fertility.In this paper the method which used for estimating the parameters of pearsonian type I curve is the last method.

4- The Application of The Models on Egypt's Data (EDHS, 1992, 1995 and 2000):

In the appendix of this paper we introduce the FORTRAN programs use to estimate the parameters of each model and calculate the estimated cumulative fertility rate in the case of Gompertz function or the estimated ASFR in the case of Hadwiger function ,pearson type III curve , and pearsonian type I curve.

4.1 Application of The Gompertz Function:

By using the FORTRAN program (1) in the appendix , we obtained the estimated parameters of Gompertz function after ten iterations (Murphy and Nagnur, 1972) and the estimated cumulative fertility rate.

Table (1) shows the estimated parameters of Gompertz function for EDHS (1992, 1995, and 2000), and Table (2) shows the estimated and observed cumulative ferlility rate for EDHS (1992, 1995, and 2000).

Table (1) shows that the values of the estimated parameters fall in the suitable range. Also we calculated the net error, using the following formula.

$$\text{Net error} = 100*\Sigma(r_i-\hat{r}_i)k_i/\Sigma r_i\ k_i$$

where r_i and r_i are the actual and estimated cumulative fertility rates by age respectively, and k_i is the standard population by age (we use the standard population suggested by Coale, 1967).

The net error for EDHS 1992 is 5.1%, for EDHS, 1995 is 4.8%, also for EDHS, 2000 is 6.3%.

Figure (1) shows the estimated ASFR using the Gompertz function for EDHS (1992, 1995, and 2000).

From figure (1) we observe that the curves of age-specific fertility rates increase monotonically to a maximum and decrease monotonically after that (the first property of Gompertz function). So we can say that the Gompertz model is suitable for Egypt's data but is not a good fit at the tails. Many authors considered the second property of Gompertz function (at the age of maximum fertility, exactly e^{-1}, about 37 percent of the total fertility will have been completed, no matter what the values of K, A, B, and t_o) is the reason for not getting good fit at the tails. (The properties of the Gompertz function discussed by Titus see Murphy and Nagnur, 1972).

Table (1)

The Estimated Parameters of Gompertz Function

TheEstimated parameter	**K**	**A**	**B**
EDHS, 1992	3.9300	0.3471	0.8442
EDHS, 1995	3.6300	0.3614	0.8377
EDHS,2000	3.5250	0.3459	0.8429

Table (2)

The Observed and the Estimated Cumulative Fertility Rate

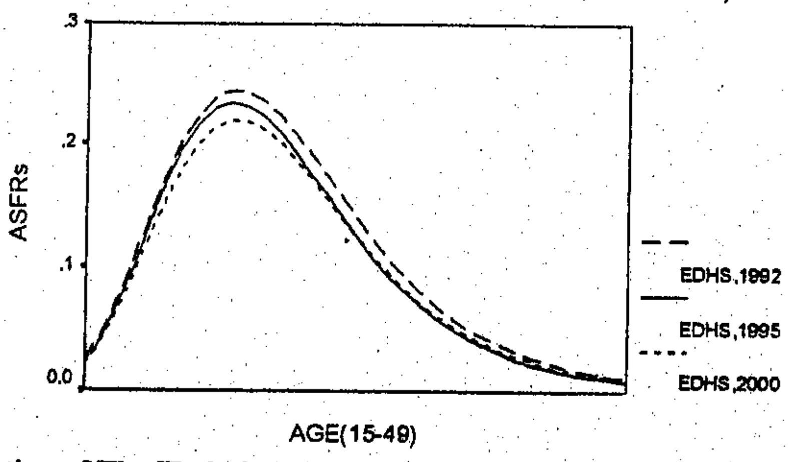
Using Gompertz Function

Age groups	**The Observed Cumulative Fertility Rates**			**The Estimated Cumulative Fertility Rates**		
	EDHS, 1992	**EDHS, 1995**	**EDHS, 2000**	**EDHS, 1992**	**EDHS, 1995**	**EDHS, 2000**
15-19	0.315	0.305	0.255	0.164	0.146	0.140
20-24	1.355	1.305	1.235	1.005	0.963	0.894
25-29	2.465	2.355	2.275	2.190	2.1	1.966
30-34	3.240	3.055	3.01	3.060	2.9	2.75
35-39	3.685	3.460	3.385	3.529	3.307	3.172
40-44	3.900	3.595	3.505	3.753	3.493	3.37
45-49	3.930	3.630	3.525	3.853	3.573	3.458

FIGURE 1

THE ESTIMATED ASFR USING THE

GOMPERTZ FUNCTION(EDHS,1992,1995,AND 2000)



4.2 Application of The Hadwiger Model:

By using the FORTRAN program (2) in the appendix , we obtained the estimated parameters of Hadwiger model and the estimated ASFR.

Table (3) shows the estimated parameters of Hadwiger model function for EDHS, 1992, 1995, and 2000, and Table(4) shows the observed and the estimated ASFRs using Hadwiger function for EDHS, 1992, 1995, and 2000.

Figures (2-4) compare between the observed and the estimated ASFRs for EDHS, 1992, 1995, and 2000 respectively.

From figures (2-4) we observe that the Hadwiger model is not suitable for Egypt's data, because there are the observed difference between the observed and estimated ASFR. Also the Hadwiger function fitted by least squares gives a poor representation of Egypt's fertility curves i.e. the loss of fit is more than 0.01.

Table (3)

The Estimated Parameters of Hadwiger Model

TheEstimated parameter	b_1	b_2	b_3
EDHS, 1992	3.66	32.11	-3.93
EDHS, 1995	3.78	32.1	-4.3
EDHS,2000	3.6	30.11	-2.08

Table (4)

The Observed and Estimated Age specific Fertility Rates Using Hadwiger Function

Age groups	The Observed ASFRs			The Estimated ASFRs		
	EDHS, 1992 TFR=.78	EDHS, 1995 TFR=.72	EDHS, 2000 TFR=.7	EDHS, 1992	EDHS, 1995	EDHS, 2000
15-19	0.063	0.051	0.051	0.056	0.055	0.045
20-24	0.208	0.200	0.196	0.197	0.192	0.183
25-29	0.222	0.210	0.208	0.251	0.236	0.236
30-34	0.155	0.140	0.147	0.168	0.149	0.151
35-39	0.089	0.081	0.075	0.074	0.061	0.061
40-44	0.043	0.027	0.024	0.024	0.018	0.018
45-49	0.006	0.007	0.004	0.006	0.005	0.004

FIGURE 2

Observed ASFR for year 1992 and

Estimated by using the simple Hadwiger model

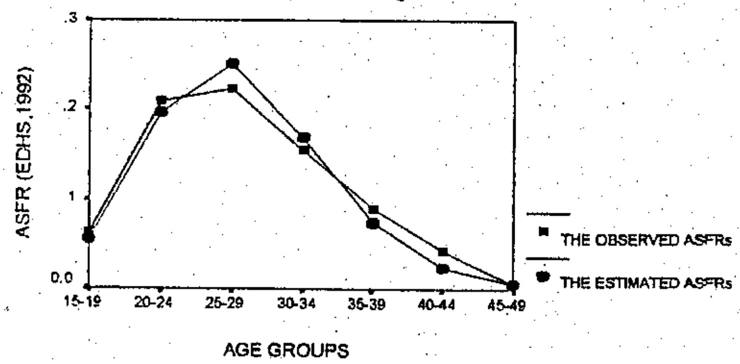


FIGURE 3

Observed ASFR for year 1995 and

Estimated by using the simple Hadwiger model

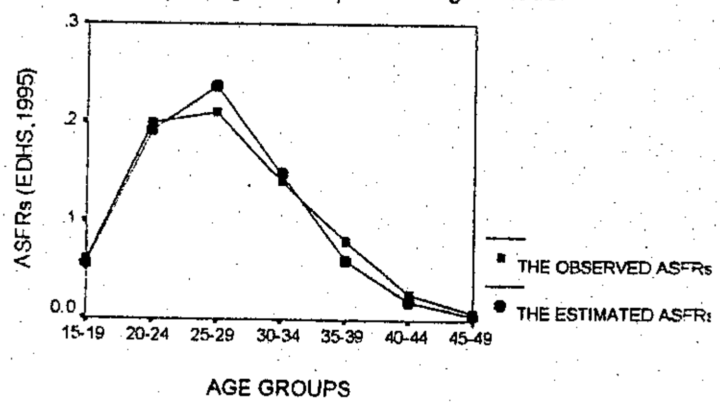
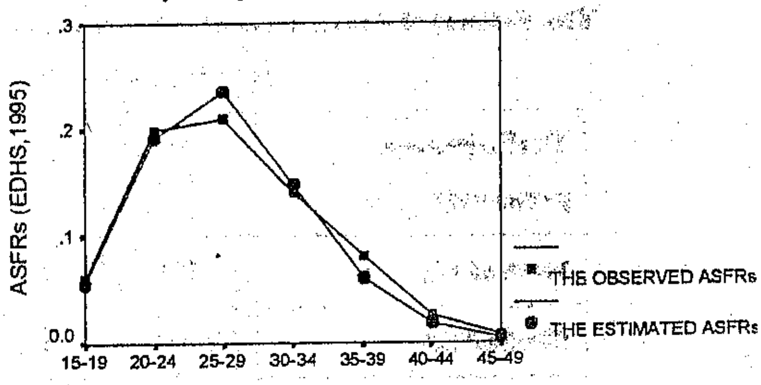


FIGURE 4

Observed ASFR for year 2000 and

Estimated by using the simple Hadwiger model



4.3 **Application of the Pearson Type III Curves**

By using FORTRAN program (3) in the appendix, we obtained the estimated parameters of Pearson type III curves and the estimated ASFR.

Tables (5-7) show the estimated parameters of the truncated Pearson type III curve for EDHS, 1992, 1995, and 2000 , and Tables (8-10) show the observed and the estimated ASFRs using the truncated Pearson type III curve for EDHS, 1992, 1995, and 2000.

Figures (5-7) compare between the observed and the estimated ASFRs for EDHS, 1992, 1995, and 2000 respectively.

From figures (5-7) we observe that the Pearson type III curve is suitable for Egypt's data, because there are not the observed differences between the observed and estimated ASFRs. Also the Pearson type III curve fitted by d (the index of differential composition or the index of dissimilarity). Where:

$$DD = 100\left(\frac{1}{2} * \frac{5d}{TFR}\right)$$

where:

d is the sum of the absolute values of the deviations of the estimated from the observed ASFR, the good data which yielding the smallest DD.Figures (5-7) show a good representation of Pearson type III curve for Egypt's fertility curves i.e.DD has small value.

Table (5)

The Estimated Parameters of the Truncated Pearson Type III

(EDHS, 1992)

TheEstimated parameter	m	u	s	MA
Method (1)	9.7751	46	15.201	28.69
Method (2)	9.2491	46	15.785	28.686
Method (3)	11.7953	46	15.078	30.26

Table (6)

The Estimated Parameters of the Truncated Pearson Type III

(EDHS, 1995)

TheEstimated parameter	m	u	s	MA
Method (1)	10.0728	43	15.072	28.347
Method (2)	9.6199	43	15.520	28.347
Method (3)	10.4793	43	15.380	28.347

Table (7)

The Estimated Parameters of the Truncated Pearson Type III

(EDHS,2000)

TheEstimated parameter	m	u	s	MA
Method (1)	9.6256	43	15.479	28.318
Method (2)	8.87781	43	16.250	28.318
Method (3)	11.5008	43	15.306	29.529

Table (8)

The Observed ASFR, and the Estimated ASFR by the Truncated Pearson Type III Curve (EDHS, 1992)

Age Group	The Observed ASFR TFR = 3.9	Estimated ASFR	Estimated ASFR	Estimated ASFR
15 – 19	0.063	0.096	0.078	0.047
20 – 24	0.208	0.203	0.208	0.161
25 – 29	0.222	0.212	0.221	0.196
30 –34	0.155	0.152	0.155	0.167
35 – 39	0.089	0.092	0.089	0.119
40 – 44	0.043	0.049	0.046	0.079
45 – 49	0.006	0.006	0.005	0.011

Table (9)

The Observed ASFR, and the Estimated ASFR by the Truncated Pearson Type III Curve (EDHS, 1995)

Age Group	The Observed ASFR TFR = 3.6	Estimated ASFR	Estimated ASFR	Estimated ASFR
15 – 19	0.061	0.062	0.068	0.057
20 – 24	0.200	0.188	0.192	0.176
25 – 29	0.210	0.199	0.205	0.200
30 –34	0.140	0.147	0.148	0.155
35 – 39	0.081	0.090	0.088	0.100
40 – 44	0.027	0.034	0.032	0.039
45 – 49	0.007	-	-	-

Table (10)

The Observed ASFR, and the Estimated ASFR by the Truncated Pearson Type III Curve (EDHS, 2000)

Age Group	The Observed ASFR TFR = 3.5	Estimated ASFR	Estimated ASFR	Estimated ASFR
15 – 19	0.051	0.063	0.078	0.047
20 – 24	0.196	0.187	0.210	0.154
25 – 29	0.208	0.199	0.200	0.188
30 –34	0.147	0.143	0.130	0.158
35 – 39	0,075	0.085	0.171	0.115
40 – 44	0.024	0.031	0.017	0.046
45 – 49	0.004	-	-	-

Figure (5)

OBSERVED ASFR ANDESTIMATED

BY TRUNCATED PEARSON TYPE III CURVE

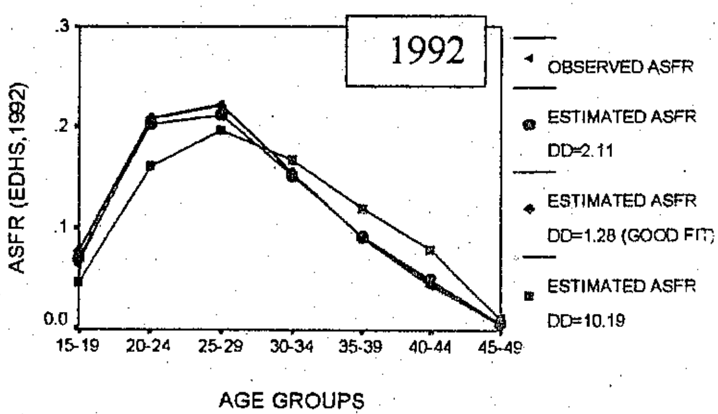
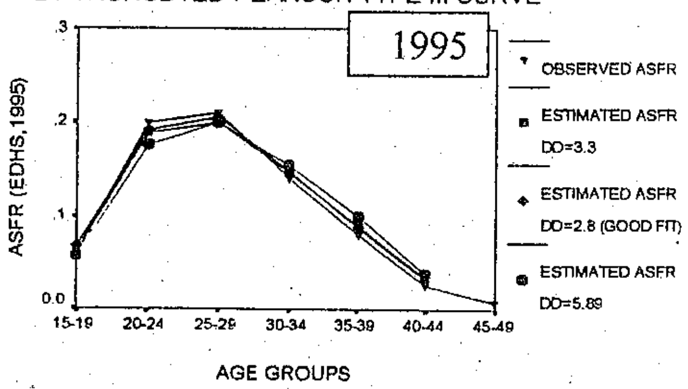


FIGURE (6)

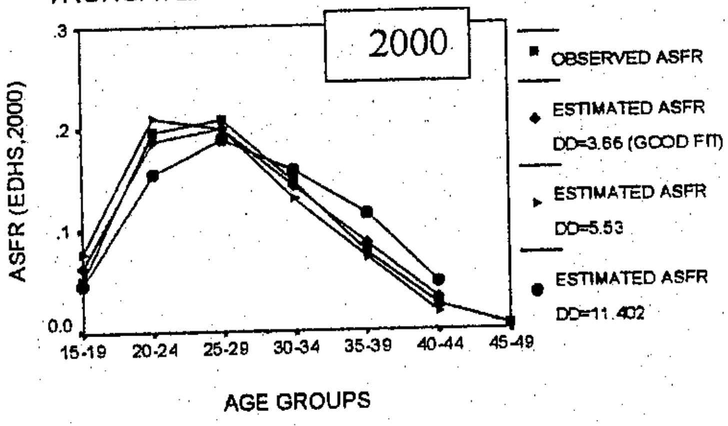
OBSERVED ASFR AND ESTIMATED

BY TRUNCDTED PEARSON TYPE III CURVE



OBSERVED ASFR AND ESTIMATED BY

TRUNCATED PEARSON TYPE III CURVE



4.4 Application of the Pearsonian Type I Curve

By using FORTRAN program (4) in the appendix, we obtained the estimated parameters of Pearsonian type I curve and the estimated ASFR.

Tables (11-12) show the results of the application of Pearsonian type I curve and comparison between the observed and the estimated ASFRs using Pearsonian Type I curve for EDHS, 1995, and 2000 because this model we can't apply this model for EDHS, 1992 (see : Mitra ,1967).

Figures (8-9) compare between the observed and the estimated ASFRs for EDHS, 1995, and 2000 respectively.

From figures (8-9) we observe that the Pearsonian Type I curve is not suitable for Egypt's data, because there are the observed differences between the observed and estimated ASFRs. Also the Pearsonian Type I curve fitted by Δ This, in percentage terms , measures the discrepancy between the observed distribution and that obtained by summing up the differences of the identical signs between the observed and model distributions by age groups ,we observe that Δ is large

Table (11)

The Results of the Application of the Type ICurve with EDHS (1995) And Comparison between Observed and Estimated ASFR.

Age Group	The Expected Values of Age-specific Fertility Rate		Observed ASFR X 1000	Δ x 100
	Relative y_i	f_i =(TFR) y_i x1000		
15-19	$1.046429X10^{-1}$	75	61	-14
20-24	$1.915202X10^{-1}$	138	200	+62
25-29	$2.139591X10^{-1}$	154	210	+56
30-34	$1.998363X10^{-1}$	144	140	-4
35-39	$1.594526X10^{-1}$	115	81	-34
40-44	$1.002946X10^{-1}$	72	27	-45
45-49	$3.169684X10^{-1}$	23	7	-16
Total	1.0000	721	721	
The total of Δ with positive signs				118
The total of Δ with minus signs				113

Table (12)

The Results of the Application of the Type I Curve with EDHS (2000) And Comparison between Observed and Estimated ASFR

Age Group	The Expected Values of Age-specific Fertility Rate		Observed ASFR X 1000	Δ x 100
	Relative y_i	f_i= (TFR) y_i x1000		
15-19	$1.040466X10^{-1}$	73	51	-22
20-24	$1.989871X10^{-1}$	140	196	+56
25-29	$2.226444X10^{-1}$	157	208	+51
30-34	$2.037592X10^{-1}$	144	147	+3
35-39	$1.559552X10^{-1}$	110	75	-35
40-44	$9.102171X10^{-2}$	64	24	-40
45-49	$2.481895X10^{-2}$	18	4	-14
Total	1.0000	706	706	
The total of Δ with positive signs				110
The total of Δ with minus signs				111

FIGURE (8)

COMPARISON BETWEEN OBSERVED

AND ESTIMATED ASFR BY (TYPE I CURVE)

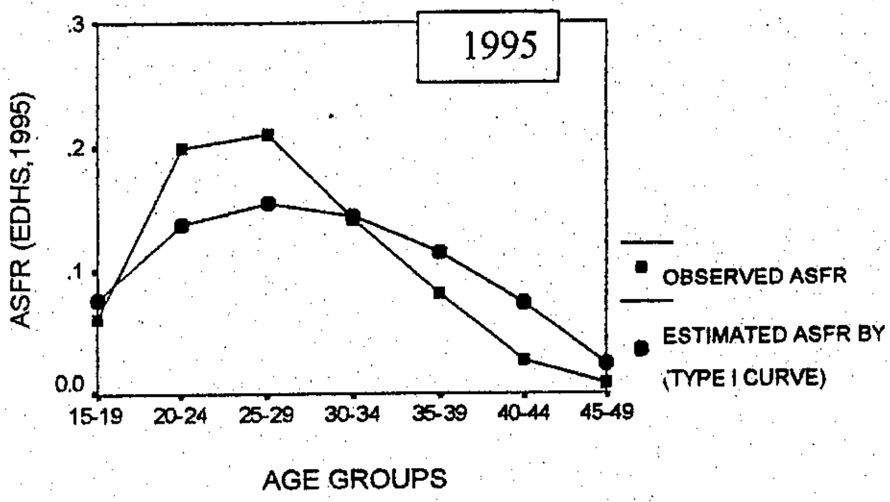
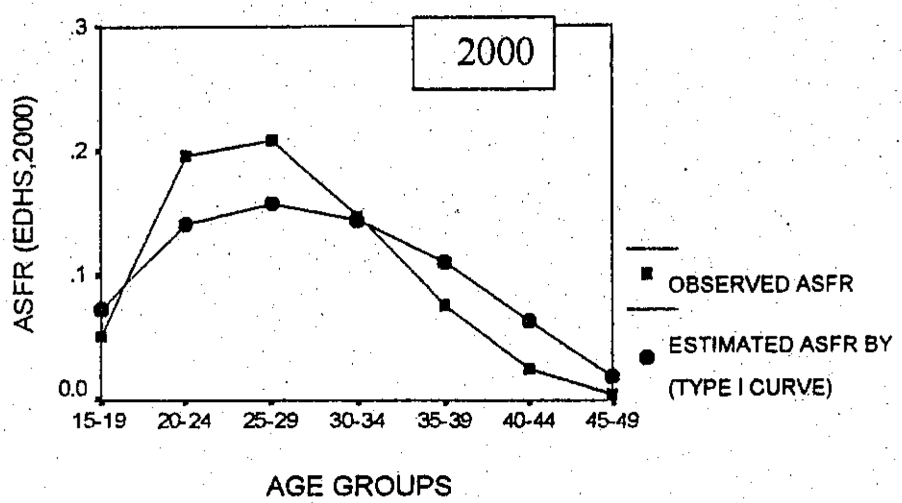


FIGURE (9)

COMPARISON BETWEEN OBSERVED

AND ESTIMATED ASFR (TYPE I CURVE)



5.1- Conclusion:

In this paper, the Gompertz model, the Hadwiger model, the Pearson type III curve , and Pearsonian type I curve were applied to the Egypt's data (EDHS, 1992, 1995 and 2000). It is observed that, the Gompertz model is suitable for Egypt's data but not at the tails. Many authers attributed this result to the second property of Gompertz function., the Hadwiger function is not suitable for Egypt's data because the loss of fit is more than 0.01, the Pearson type III curves is more suitable for Egypt's data than other models because DD has small values, but the Pearsonian type I curve is not suitable for Egypt's data because Δ has large value. Hence, the Pearson type III curves is the suitable fertility model in Egypt .

5.2 Recommendations:

From the above analysis, the following can be recommended :

(1) Researches have to be made for searching for a new mathematical function for graduating the age – specific fertility rate.

(2) Researches have to made for using the mixture function as a mathematical models of fertility in Egypt.

(3) Researches have to be made for using a new methods for estimating the parameters of the mathematical models which applied in this demographic study to get more accurate estimated values of the parameters.

(4) Efforts have to be made for improving the quality of data collection and compilation processes so that estimates of demographic indicators could be more accurate.

(5) Efforts have to be made for providing the EDHS data with age – specific fertility rate by single year age of mothers to apply the mathematical models more accurate.

APPENDIX

PROGRAM(1)
THE GOMPERTZ FUNCTION

```
C-----N.L=THE DIMENTIONS OF MATRICES
C-----a0,b0=THE INITIAL VALUES OF GOMPERTZ'S PARAMETERS
C-----PK=TOTAL FERTILITY RATE
C-----t0=THE NEW ORIGIN
C-----mm=THE NUMBER OF ITERATION WHICH EQUAL 10(MURPHY-
NAGNUR,1972)
      dimension a(2,2),b(2,2),c(2,1),h(2,1),p(7),y(7)
      open(5.file='data32')
      open(6.file='output32')
      read(5,*)n.l.a0.b0,pk,t0,mm
      do 556 i=1.7
556   read(5.*)p(i)
444   do 555 i=1.mm
      t=17.5
      w1=0.0
      w22=0.0
      w4=0.0
      w6=0.0
      w8=0.0
      do 222 j=1.7
      y(j)=pk*a0**(b0**(t-t0))
      w1=w1+(y(j)-p(j))*y(j)*(b0**(t-t0))
      w22=w22+((y(j)-p(j))*y(j)*(t-t0)*(b0**(t-t0)))
      w4=w4+pk*(y(j)**2)*((b0**(t-t0))**2)+((y(j)-p(j))
     1*((b0**(t-t0))**2)*y(j))-((y(j)-p(j))*y(j)*(b0**(t-t0)))
      w6=w6+pk*((y(j)*(t-t0)*(b0**(t-t0))*alog(a0))**2)
     1+((y(j)-p(j))*y(j)*((t*(b0**(t-t0))*alog(a0))**2))
     1+((y(j)-p(j))*y(j)*((t-t0)**2)*(b0**(t-t0))*alog(a0))
     1-((y(j)-p(j))*y(j)*(t-t0)*(b0**(t-t0))*alog(a0))
      w8=w8+((y(j)-p(j))*y(j)*(t-t0)*(b0**(t-t0)))
     1+((y(j)-p(j))*((b0**(t-t0))**2)*(t-t0)*alog(a0)
     1*y(j))+((y(j)**2)*(t-t0)*alog(a0))
      t=t+5
222   continue
      w2=(2*pk*w1)/a0
      w3=(2*pk*w22*alog(a0))/b0
      w5=(2*pk*w4)/(a0**2)
      w7=(2*w6*pk)/(b0**2)
      w9=(2*pk*w8)/(a0*b0)
      a(1,1)=w5
      a(1,2)=w9
      a(2,1)=w9
      a(2,2)=w7
      h(1,1)=w2
      h(2,1)=w3
```

```
      call inv (a,n,l,b,ifail)
      if(ifail.eq.1.0) go to 55
      do 100 ii=1,n
      s=0.0
      do 300 k=1,l
      s=s+b(ii,k)*h(k,1)
 300  continue
      c(ii,1)=-s
 100  continue
      a0=a0+c(1,1)
      b0=b0+c(2,1)
      write(6,*)'the estimated values of the constants a0,b0'
      write(6,*)i,a0,b0
 555  continue
      t=17.5
 855  yy=pk*(a0**(b0**(t-t0)))
      write(6,*)'the age of mother','the estimated value of CFR'
      write(6,*)t,yy
      t=t+5
      if(t.lt.50) go to 855
      stop
 55   write(6,*)'no inv'
      stop
      end
      subroutine inv (a,n,l,b,ifail)
      dimension a(n,l),b(n,l)
      ifail=0
      do 1 i=1,n
      do 2 jj=1,l
      if(jj.eq.i) then
      b(i,jj)=1.0
      else
      b(i,jj)=0.0
      end if
 2    continue
 1    continue
      do 3 k=1,n
      if(k.eq.n) go to 4
      call find (a,n,k,l,j)
      if(j.eq.k) go to 4
      call change (a,n,k,l,j)
      call change (b,n,k,l,j)
 4    if(a(k,k).eq.0.0) go to 5
      t=a(k,k)
      do 6 i=1,l
      a(k,i)=a(k,i)/t
 6    b(k,i)=b(k,i)/t
      do 7 i=1,n
      if(i.eq.k) go to 7
      d=-a(i,k)
```

```
   call row (a,n,k,l,i,d)
   call row (b,n,k,l,i,d)
 7 continue
 3 continue
   return
 5 ifail=1
   return
   end
   subroutine find (a,n,k,l,j)
   dimension a(n,l)
   amax=abs(a(k,k))
   j=k
   do 11 i=k+1,n
   if(abs(a(i,k)).le.amax) go to 11
   amax = abs(a(i,k))
   j=i
11 continue
   return
   end
   subroutine change (a,n,k,l,j)
   dimension a(n,l)
   do 31 i=k,l
   aa=a(k,i)
   a(k,i)=a(j,i)
   a(j,i)=aa
31 continue
   return
   end
   subroutine row (a,n,k,l,j,d)
   dimension a(n,l)
   do 41 i=1,l
41 a(j,i)=a(j,i)+a(k,i)*d
   return
   end
```

PROGRAM(2)
THE HADWIGER MODEL

```
C-----MEAN OF AGE (AM)
C-----VARIANCE OF AGE (V)
C-----MODE OF AGE (CMO)
C-----INITIAL VALUE OF B3
C-----TOTAL FERTILITY RATE (TFR)
C-----RELATIVE AGE-SPECIFIC FERTILITY RATE (RASFR)
C-----AGE-SPECIFIC FERTILITY RATE (ASFR)
      open(5,file='data11')
      open(6,file='output11')
      read(5,*)d0.am.v.cmo.tfr
      call seq(b0.am.v.cmo.b1.b2)
      x=15
   10 y1=ff(x,b0.b1.b2)
      y2=ff(x+1.25.b0,b1,b2)
      y3=ff(x+2.5.b0.b1.b2)
      y4=ff(x+3.75.b0,b1,b2)
      y5=ff(x+5.b0.b1.b2)
      rasfr=.417*(y1+4*y2+2*y3+4*y4+y5)
      asfr=tfr*rasfr
      write(6,*)rasfr,asfr
      x=x+5
      if(x.le.45) go to 10
      stop
      end
      subroutine seq(b0,am,v,cmo,b1.b2)
      do 1 i=1.10
      b2=am-b0
      b1=(am-b0)/((2*v)**.5)
      b3=cmo-b2*(((1+(16/9)*(b1**4))**.5)-1)/((4/3)*(b1**2))
      b0=b3
      write(6,*)b0
    1 continue
      write(6.*)b1.b2
      return
      end
      function ff(x.b0.b1,b2)
      ff=(b1/(1.77*b2))*((b2/(x-b0))**1.5)*exp(-(b1**2)
     1*((b2/(x-b0))+((x-b0)/b2)-2))
      return
      end
```

program(3)
The Truncated Pearson type III Curve
Incomplete Gamma Distibution)

```
C----U=THE FINAL AGE OF FERTILITY(THE POINT OF TRUNCATION)
C-----S=THE BEGINNING AGE OF FERTILITY
C-----T=TOTAL FERTILITY RATE
C-----N=THE LENGTH OF THE AGE INTERVAL(N=5)
C-----(K,L)=(15,45)
C-----AM=THE MEAN(MA)
C-----NN=THE NUMBER OF ITERATION
      open(5,file='data111')
      open(6,file='output111')
      read(5,*)u,s,t,n,k,l,nn,am
      call mkh(u,s,nn,am,y)
      r=4*t
      fr=(y**2)-2*exp(-2*(u-s)/y)*(((u-s)**2)+y*(u-
      s)+.5*(y**2))
      pk=r/(y*fr)
      es=0.0
      x=k
 1    if(x.ge.s) then
      go to 10
      else
      x=s
      end if
 10   d=x+n
      if(u.ge.d) then
      go to 20
      else
      d=u
      u=d
      end if
 20   z=((pk*y)/(2*n))*(f(x,s,y)-h(d,s,y))
      es=es+z
      write(6,*)'the estimated ASFR'
      write(6,*)z
      write(6,*)x,d,s,u
      x=int(x)
      x=x+n
      if(x.le.l) then
      go to 1
      else
      write(6,*)'the GTF'
      write(6,*)es
      write(6,*)'the value of k','the value of m'
      write(6,*)pk,y
      end if
      stop
```

```
      end
      subroutine mkh(u,s,nn,am,y)
      y1=am-.85*s-5.75
      y2=(.6667)*(am-s)
      gm0=amax1(y1,y2)
      do 8 j=1,nn
      y=gm0-(ff(gm0,u,s,am)/gg(gm0,u,s,am))
      if(y.eq.gm0) then
      write(6,*)y
      go to 22
      else
      gm0=y
      end if
8     continue
22    return
      end
      function ff(x,u,s,am)
      a=(s-am)*(((u-s)**2)+(u-s)*x+.5*(x**2)-
      .5*(x**2)*exp(2*(u-s)/x))
      b=((u-s)**3)+1.5*((u-s)**2)*x+1.5*(u-
      s)*(x**2)+.75*(x**3)-.75*
     1(x**3)*exp(2*(u-s)/x)
      ff=a+b
      return
      end
      function gg(x,u,s,am)
      c=(s-am)*((u-s)+x-x*exp(2*(u-s)/x)+(u-s)*exp(2*(u-s)/x))
      d=1.5*((u-s)**2)+3*(u-s)*x+2.25*(x**2)-2.25*(x**2
     1*exp(2*(u-s)/x)+1.5*(u-s)*x*exp(2*(u-s)/x)
      gg=c-d
      return
      end
      function f(x,s,y)
      f=((x-s)**2)+y*(x-s)+.5*(y**2))*exp(-2*(x-s)/y)
      return
      end
      function h(yy,s,y)
      h=((yy-s)**2)+y*(yy-s)+.5*(y**2))*exp(-2*(yy-s)/y
      return
      end
```

PROGRAM (4)

THE PEARSONIN TYPE I CURVE (BETA DISTRIBUTION)

```
C---H=THE NUMBER OF WOMEN ARE STILL MARRIED IN AGE GROUP 20-24
C---E=THE NUMBER OF MOMEN ARE STILL MARRIED IN AGE GROUP 25-29
C-----T=THE TOTAL NUMBER OF WOMEN ARE STILL MARRIED
C-----GTF=THE GROSS TOTAL FERTILITY RATE
      f(x,n,r,Z,v)=(((1+x/n)**r)*((1-x/(35-n))**Z))/v
      open(5,file='data',status='old')
      open(6,file='output1')
      read(5,*)H,E,T,GTF
      CALL MODAL(H,E,T,R,Z,FN,p,v1,AM,N)
      if(p.gt.8.0) go to 10
      if((v1.lt.28.1).AND.(v1.gt.30.0)) go to 10
      call per(n,r,Z,v)
      regtf=0.0
      G=0.0
      do 1 k=-n,30-n,5
      a=f(k+1.25,n,r,Z,v)
      b=f(k+2*1.25,n,R,Z,v)
      c=f(k+3*1.25,n,r,Z,v)
      s=f(k,n,r,Z,v)+f(k+5,n,r,Z,v)+4*(a+c)+2*b
      d=(1.25*s)/3
      regtf=regtf+d
      EASFR=GTF*D
      G=G+EASFR
      write(6,*)'expected value of REASFR' ,'ESTIMATED ASFR'
      write(6,*)d,EASFR
    1 continue
      write(6,*)'sum REGTF','SUM EASFR'
      write(6,*)REGTF ,G
      GO TO 30
   10 write(6,*)'this model not suitable'
   30 stop
       end
      subroutine per(n,r,Z,v)
      u(x)=(((1+x/n)**r)*((1-x/(35-n))**z))
      v=0.0
      do 2 j=-n,35-n
      v=v+u(j)
    2 continue
      write(6,*)'sum'
      write(6,*)v
      return
      end
      SUBROUTINE MODAL(H,E,T,R,Z,FN,p,v1,AM,N)
      R1=H/T
      R2=E/T
      P=100*(R2-R1)/(5*R1)
      AM=P+22
      FN=AM-15
```

```
    N=FN
    V1=AM+2.5
    R=((AM-15)*(35-2*(V1-15)))/(35*(V1-AM))
    Z=((35-FN)*R)/FN
    WRITE(6,100)R,Z,AM,FN,p,v1,N
100 format(1x,f10.6,f10.6,1x,F10.6,F10.6,1x,f10.6,f10.6,I6)
    RETURN
    END
```

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