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Multivariate Product Type Of Estimator And Its Applications In Family Planning Programs

By

Mounira Ahmed Hussein*

* Associate professor in the faculty of commerce Menofia University, Shebein El koum, Egypt.

Summary:

In this paper, a multivariate product type of estimator is suggested for positively correlated auxiliary variables in a finite population.

This estimator is used for estimating the prevalence rate at a given value of socio – economic and family planning variables. This estimator is always better than the mean per element.

I. Introduction:

Multivariate product type of estimators, are proposed for simple random sample by (Hussein, 1988). These estimators are better than the mean per element for some region (Hussein, 1988, 1995). The new modified estimator is always better than the mean per element. It is also superior to the ratio and the original product estimator for other regions. The author had also suggested two estimators for the median in the presence of two auxiliary variables (Hussein, 2001).

II. Notations:

1- N and n < N are the population and sample size.

2- $\bar{Y}$ and $\bar{X}$ are the population means for the characteristic under study and the auxiliary variables respectively.

3- $\bar{y}$ and $\bar{\chi}$ are the sample means of the characteristic under study and the auxiliary variables respectively.

4- $S_Y^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (Y_i - \bar{Y})^2$ is the variance for the population of the characteristic under study:

5- $S_{yx} = \frac{1}{(N-1)} \sum_{i=1}^{N} (Y_i - \bar{Y})\ (X_i - \bar{X})$ is the covariance between Y_i and X_i

6- $C_{yy} = \frac{S_Y^2}{\bar{Y}^2}$ is the square of the coefficient of variation of Y or the relative variance of Y.

7- $C_{xx} = \frac{S_x^2}{\bar{X}^2}$ is the square of the coefficient of variation of X or the relative variance of X.

8- $C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$ is the relative covariance.

9- $\hat{Y} = N\bar{Y}$, $\hat{X} = N\bar{x}$ are unbiased estimators of Y and X based on a sample of size n.

10- Suppose that Y_i follows a simple linear model.

$$Y_i = a + b_i\chi_i + e_{ij} \qquad i = 1, \ldots\ldots, N$$

$$for\ j = 1, \ldots\ldots, P$$

Where a, b are fixed constant for a given (J) e_i are random errors independent of the χ_i with mean zero and $\text{var}(\chi_{ij}) = \sigma^2$ exists.

III: Multivariate product type of estimator:

An univariate product estimator was proposed independently by servenkataraman (1980) and Bandyopadhyay (1980). The author had suggested multivariate version for simple and another versions for stratified sample and cluster sample combined with stratification (Hussein, 1988, 1995, 1999).

The univariate version can be presented as follows:

$$\hat{Y}_a = \hat{X}^{*}\frac{\hat{Y}}{X}$$

Where

$$\hat{X}^{*} = \frac{NX - n\hat{X}}{(N-n)}$$

Where

$$\hat{X} = N\bar{x}$$

and $\quad X = N\bar{X}$

$$\therefore \hat{Y}_a = \frac{(NX - n\hat{X})\hat{Y}}{(N-n)X}$$

$$= \frac{N}{(N-n)}\hat{Y} - \frac{n}{(N-n)}\frac{\hat{X}\hat{Y}}{X}$$

let $g=\frac{n}{N-n}$

$$\therefore \hat{Y}_a=(1+g)N\bar{y}-gN\bar{y}\frac{\bar{\chi}}{\bar{X}}$$

$$\therefore \hat{\bar{Y}}_a=(1+g)\bar{y}-g\bar{y}\frac{\bar{\chi}}{\bar{X}}$$

From a model based standpoint, it is helpful to consider a slight expansion of the linear model used by Cochran (1977), Dorfman (1994), Rao and Shao (1996) to construct the new modified estimator.

We are proposing the following estimator

$$Y_{Ma}=(1+g)\bar{y}-g\bar{y}\frac{a+\sum bj\bar{\chi}_j}{a+\sum bj\bar{X}_j}$$

as an estimator based on linear combination of the auxiliary variable instead of the mean per element.

Assuming that

$$a+\sum_{j=1}^{p} bj\bar{\chi}_j=1$$

$$\therefore \bar{Y}_{Ma}=(1+g)\bar{y}-g\bar{y}\frac{1}{1-\sum b_j\bar{\chi}_j+\sum b_j\bar{X}_j}$$

$$\bar{Y}_{Ma}=(1+g)\bar{y}-\frac{g\bar{y}}{1-\sum b_j(\bar{\chi}_j-\bar{X})}$$

$$\bar{Y}_{Ma}=(1+g)\bar{y}-\frac{g\bar{y}}{1-\sum K_j\frac{(\bar{\chi}_j-\bar{X}_j)}{\bar{X}_j}}$$

$$\therefore \bar{Y}_{Ma}=(1+g)\bar{y}-g\bar{y}\left[1-\sum K_j\frac{(\bar{\chi}_j-\bar{X}_j)}{\bar{X}_j}\right]^{-1}$$

The univariate version of this estimator can be presented as follows.

$$\bar{Y}_{ma}=(1+g)\bar{y}-g\bar{y}\left[1-K\frac{(\bar{\chi}-\bar{X})}{X}\right]^{-1}$$

IV The mean square error of the proposed multivariate product type of estimator.

$$\Theta\ \bar{Y}_{Ma} = (1+g)\bar{y} - g\bar{y}\left[1 - \sum Kj\frac{(\bar{\chi}_j - \bar{X})}{\bar{X}_j}\right]^{-1}$$

Let $\bar{y} = \bar{Y}(1+e)$

Where $e = \dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$

And $\bar{\chi}_j = \bar{X}_j(1+e_j)$

Where $e_j = \dfrac{\bar{\chi}_j - \bar{X}_j}{\bar{X}_j}$

$$\bar{Y}_{Ma} = (1+g)\bar{Y}(1+e) - g\bar{Y}(1+e)\ \ (1-\sum K_j e_j)^{-1}$$

$$\bar{Y}_{Ma} = (1+g)\bar{Y}(1+e) - g\bar{Y}(1+e)\left[1+\sum K_j e_j + (\sum K_j e_j)^2 + \ldots\ldots\ldots\ldots\right.$$

$$= \bar{Y} + g\bar{Y} + e\bar{Y} + ge\bar{Y}$$

$$-g\bar{Y}\left[1+\sum K_j e_j + (\sum K_j e_j)^2 + e + \sum K_j ee_j + e(\sum k_j e_j)^2\right.$$

$$(\bar{Y}_{Ma} - \bar{Y}) \approx e\bar{Y} - g\bar{Y}\left[\sum K_j e_j + \left(\sum K_j e_j\right)^2 + \sum K_j ee_j + \ldots\ldots\right]$$

$$\therefore (\bar{Y}_{Ma} - \bar{Y})^2 \approx e^2\bar{Y}^2 + g^2\bar{Y}^2\left[(\sum K_j e_j)^2 + (\sum K_j e_j)^4 + (\sum K_j ee_j)^2 + \ldots\right]$$

$$-2g\bar{Y}^2\sum K_j ee_j + \ldots\ldots\ldots$$

$$\therefore E(\bar{Y}_{Ma} - \bar{Y})^2 \approx \bar{Y}^2 E(e^2) + g^2\bar{Y}^2 E\sum_j\sum_{j'} K_j K_{j'}(e_j e_{j'}) - 2g\bar{Y}^2\sum K_j E(ee_j)$$

$$M(\bar{Y}_{Ma}) \approx \bar{Y}^2 C_{yy} - 2g\bar{Y}^2\sum_d K_j C_{YX_j} + g^2\sum_j\sum_{j_1} K_j K_{j'} C_{X_j X_{j'}}$$

We can express the mean square error in matrix form as follows: Assuming we have only one auxiliary variable.

$M(\bar{Y}_{Ma} = \bar{Y}^2\ KGK$

where, $K = [1 \quad K]$

$$G = \begin{bmatrix} C_{yy} & -gC_{yx} \\ -gC_{yx} & g^2 C_{xx} \end{bmatrix}$$

$$M(\bar{Y}_{Ma}) = [1 \quad K]\begin{bmatrix} C_{yy} & -gC_{yx} \\ -gC_{yx} & g^2 C_{xx} \end{bmatrix}\begin{bmatrix} 1 \\ K \end{bmatrix}$$

$$= \bar{Y}^{-2}\left[C_{yy} - gKC_{yx} - gKC_{yx} + g^2 K^2 C_{xx}\right]$$

$$\therefore M(\bar{Y}_{Ma}) = \left[C_{yy} - 2gKC_{yx} + g^2 K^2 C_{xx}\right]$$

where k is a constant that minimize the mean square error.
To get the value of k we proceed as follows:

$$\frac{\partial M(\bar{Y}_{Ma})}{\partial K} = -2gC_{yx} + 2g^2 kC_{xx} \overset{set}{=} 0$$

$$\therefore 2gC_{yx} = 2g^2 KC_{xx}$$

$$\therefore K = \frac{C_{yx}}{gC_{xx}}$$

Assuming we have two auxiliary variables.

$$M(\bar{Y}_{Ma}) = \bar{Y}\, \underset{\approx}{K}\, G\, \underset{\approx}{K}'$$

Where $\underset{\approx}{K} = [1 \quad K_1 \quad K_2]$

$$G = \begin{bmatrix} C_{yy} & -gC_{yx_1} & -gC_{yx_2} \\ -gC_{yx_1} & g^2 C_{x_1x_1} & g^2 C_{x_1x_2} \\ -gC_{yx_2} & g^2 C_{x_2x_1} & g^2 C_{x_2x_2} \end{bmatrix}$$

$$M(\bar{Y}_{Ma}) = \bar{Y}^{-2}[\, C_{yy} - gK_1 C_{yx_1} - gK_2 C_{yx_2} - gK_1 C_{yx_1} + K_1^2 g^2 C_{x_1x_1} + K_1 K_2 g^2 C_{X_1X_2} - gK_{2C_{YX_2}} + K_1K_2 g^2 C_{X_1X_2} + K_2^2 g^2 C_{X_2X_2}\ ]$$

$$\frac{\partial M}{\partial k_1}(\bar{Y}_{Ma}) = -gC_{yx_1} - gC_{yx_1} + 2K_1 g^2 C_{x_1x_1} + K_2 g^2 C_{x_1x_2} + K_2 g^2 C_{x_1x_2} \ \underset{=}{set}\, 0$$

$$\therefore -2gC_{yx_1} + 2K_1 g^2 C_{x_1x_1} + 2K_2 g^2 C_{x_1x_2} \overset{set}{=} 0$$

$$\therefore -2gC_{yx_1} + 2K_1 g^2 C_{x_1x_1} + 2K_2 g^2 C_{x_1x_2} = 0 \rightarrow (1)$$

$$\frac{\partial M}{\partial k_2}(\bar{Y}_{Ma}) = -2gC_{yx_2} + 2K_1 g^2 C_{x_1x_2} + 2K_2 g^2 C_{x_2x_2} \ \underset{=}{set}\, 0$$

$$\therefore -gC_{yx_2} + K_1 g^2 C_{x_1x_2} + K_2 g^2 C_{x_2x_2} = 0 \rightarrow 2$$

Adding equation (1) and (2) we get the following.

$$-g\left[C_{yx_1}+C_{yx_2}\right]+K_1\left[g^2 C_{x_1x_1}+g^2 C_{x_1x_2}\right]+K_2\left[g^2 C_{x_1x_2}+g^2 C_{x_2x_2}\right]=0$$

$$\therefore K_1=\frac{C_{yx_1}}{g\left[C_{x_1x_1}+C_{x_1x_2}\right]}$$

$$\therefore K_2=\frac{C_{yx_2}}{g\left[C_{x_2x_2}+C_{x_1x_2}\right]}$$

Assuming we have three auxiliary variables.

$$M(\bar{Y}_{Ma})=\bar{Y}^2\, \underset{\approx}{K} G \underset{\approx}{K}^1$$

Where $\underset{\approx}{K}=[1 \quad K_1 \quad K_2 \quad K_3]$,

$$G=\begin{bmatrix} C_{yy} & -gC_{yx_1} & -gC_{yx_2} & -gC_{yx_3} \\ -gC_{yx_1} & g^2C_{x_1x_1} & g^2C_{x_1x_2} & g^2C_{x_1x_3} \\ -gC_{yx_2} & g^2C_{x_2x_1} & g^2C_{x_2x_2} & g^2C_{x_2x_3} \\ -gC_{yx_3} & g^2C_{x_3x_1} & g^2C_{x_3x_2} & g^2C_{x_3x_3} \end{bmatrix}$$

$$\therefore M(\bar{Y}_{Ma})=C_{yy}-2gK_1C_{yx_1}-2gK_2\,C_{yx_2}-2gK_3\,C_{yx_3}+K_1^2 g^2\,C_{x_1x_1}+K_2^2 g^2\,C_{x_2x_2}$$
$$+K_3^2\,g^2\,C_{x_3x_3}+2\,K_1K_2\,g^2\,C_{x_1x_2}+2K_1K_3g^2\,C_{x_1x_3}+2K_2K_3\,C_{x_2x_3}$$

$$\frac{\partial}{\partial K_1}M(\bar{Y}_{Ma})=-2g\,C_{yx_1}+2K_1g^2C_{x_1x_1}+2K_2\,g^2\,C_{x_2x_1}+2K_3g^2C_{x_1x_3} \quad = set\, 0$$

$$\frac{\partial}{\partial K_2}(\bar{Y}_{Ma})=-2gC_{yx_2}+2K_2g^2C_{x_2x_1}+2K_1g^2C_{x_2x_1}+2K_3g^2C_{x_2x_3}=set\, 0$$

$$\frac{\partial}{\partial K_3}M(\bar{Y}_{Ma})=-2gC_{yx_3}+2K_3g^2C_{x_3x_3}+2K_2C_{x_2x_2}+2K_1g^2C_{x_1x_3} \quad = set\, 0$$

Adding the three equations and solving for k_1, k_2 and k_3 we get the following:

$$K_1=\frac{C_{yx_1}}{g\,(C_{x_1x_1}+C_{x_1x_2}+C_{x_1x_3})}, \qquad K_2=\frac{C_{yx_2}}{g\,(C_{x_2x_1}+C_{x_2x_2}+C_{x_2x_3})}$$

$$K_3=\frac{C_{yx_3}}{g\,(C_{x_3x_1}+C_{x_3x_2}+C_{x_3x_3})},$$

$$\therefore K_j=\frac{C_{yx_j}}{g\sum_{j=1}^{p} C_{x_j\,x_{j1}}}$$

II. Comparison of the proposed estimator in the presence of one auxiliary variable with that of its original version and the usual ratio estimator.

a- The mean square of the proposed estimator compared to the mean per element.

$M(\bar{Y}_{Ma}) < (\bar{Y})$ *iff*

$C_{yy} - gK\,C_{yx} + g^2K^2C_{xx} < C_{yy}$ *iff* $1 < 2$

Which is always satisfied so, $\bar{Y}_{ua}$ is always better than $\bar{Y}$.

b- The mean square error of the proposed estimator compared to the original version.

iff $M(\bar{Y}_{ua}) < M(\bar{Y}_a)$

$$C_{yy} - 2gKC_{yx} + g^2K^2C_{xx} < C_{yy} - 2gC_{yx} + g^2C_{xx}$$

iff $2g(1-K)C_{yx} < g^2(1-K)^2\,C_{xx}$

iff $\dfrac{C_{yx}}{C_{xx}} < g$

iff $2\dfrac{C_{yx}}{C_{xx}} < 2g$

c- The mean square error of the original product type of estimator compared to the usual ratio estimator.

$M(\bar{Y}_a) < M(\bar{Y}_R)$

iff $C_{yy} - 2gC_{yx} + g^2C_{xx} < C_{yy} - 2C_{yx} + C_{xx}$.

iff $2(1-g)C_{yx} < (1-g^2)C_{xx}$

iff $2\dfrac{C_{yx}}{C_{xx}} < 1 + g$

So, we can conclude the following:

1-The new proposed estimator is always better than the mean per element.

2-The new proposed estimator is better than the original version of the product type estimator and the usual ratio estimator.

iff $2\dfrac{C_{yx}}{C_{xx}} < 2g$

3-The original version of the product type estimator is the better than the new proposed estimator and the usual ratio estimator.

$$iff \quad 2g < \frac{2C_{yx}}{Cxx} < (1+g).$$

4-The usual ratio estimator is better than the proposed estimator and the original one.

$$iff \quad 2\frac{C_{yx}}{C_{xx}} > (1+g).$$

When we have one auxiliary variable we have to choose between the estimators according to the preceding criterion.

Numerical example:

It is believed now that socio - economic development and family planning programs have both significant roles in bringing about fertility decline (Khalifa, Moneim & Zohry, 1999). Also it is believed that socio economic development affects the performance of the family planning programs and that family planning effort can influence socio economic development. Nine variables for family planning and seven variables for socio economic development have been selected by Khalifa, Moneim & Zohry (1994), it is found that three of the family planning variables and the seven socio economic variables have significant correlation with the contraceptive prevalence rate. A simple random sample (approximately by 50%) from governorates is chosen and the data for this sample is used to estimate contraceptive prevalence rate.

Table (1) shows the sample means and their coefficients of variation and the population means. Table (2) also shows correlation coefficients with the contraceptive prevalence rate and its significant level.

Table(1): Sample means and the coefficients of variation and the population means.

Se	Population means	Sample means	Coefficient of variation	Population means
1	Number of women per family planning center (x_1) per year	2068.08	0.3992	2099.57
2	Number of women per pharmacy (x_2)	820.80	0.3126	809.14
3	Percentage contribution of the family of the future in the distribution of contraceptives (x_3)	26.71	0.2752	30.37
4	literacy rate for population 10 years and more (x_4)	47.48	0.2056	49.77
5	Primary and secondary school enrolment (x_5)	75.06	0.1188	76.89
6	Life expectancy at birth(x_6)	63.20	0.0282	63.43
7	Infant mortality rate (x_7)	41.54	0.3175	39.24
8	Per capita Income (x_8)	902.80	0.2359	994.77
9	Percent working in agriculture (x_9)	41.52	0.3175	38.33
10	Percent urban (x_{10})	35.56	0.6246	43.03
11	The contraceptive prevalence rate (Y)	44.05	0.3292	45.4

*: Source; Khalifa M.A., Moneim A.A. and Zohry, A.G.(1994).

Table (2): The correlation coefficients between the contraceptive prevalence rate and other variables

1- Number of women per family planning center (x_1)	0.4519 P= 0.040
2-Number of women per pharmacy (x_2)	-0.5715 P= .007
3- Percentage contribution of the family of the future in the distribution of contraceptives (x_3)	0.5657 P= 0.008
4- Lit racy rate for population 10 years and more (x_4)	0.8077 P= .000
5- Primary and secondary school enrolment (x_5)	0.7524 P= 0.000
6- Life expectancy at Birth (x_6)	0.8694 P=0.000
7- Infant Mortality rate (x_7)	-0.7310 P= .000
8- Per capita income (x_8)	0.5469 P=0.010
9- Percent working in agriculture (x_9)	-0.6857 P= 0.001
10- Percent urban (x_{10})	0.5563 P= 0.009

Table (3)
The coefficients of variation

	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Y	0.10835	0.06303	-.07801	0.03402	0.05177	0.02694	0.008199	-.07687	.03601	-.08216	.09123
X_1		.15939	-.07702	.06436	.04745	.01029	.003946	-.03654	.02524	-.11687	.20913
X_2			.09773	-.02904	.03499	-.01623	-.007089	.07318	-.03533	.05458	-.0892
X_3				.07572	.02563	.00977	.002572	.03564	.03979	-.06603	.08986
X_4					.04225	.02156	.00417	-.02591	.02182	-.07598	.09557
X_5						.01412	.00258	-.01633	.01196	-.03507	.03559
X_6							.00079	-.00731	.00393	-.00623	.00656
X_7								.10082	.05887	.03551	-.0427
X_8									.05565	-.03897	.059399
X_9										.15811	-.02058
X_{10}											.3909

From table (2) we can notice that seven variables have positive significant correlations while three only have negative significant correlations.

Applying the criteria for choosing between estimators we get the following results.

Table (4) **The best estimator for each auxiliary variable**

The variable	The criteria*	The best estimator
Number of women per family planning	$2\frac{C_{yx_1}}{C_{x_1x_1}}=0.79089<2g$	The proposed estimator
Percentage contribution of the family of the future in the distribution of contraceptives (x_3)	$2\frac{C_{yx_4}}{Cx_4x_4}=0.89857<2g$	The proposed estimator
Lit racy rate for population 10 years and more (x_4)	$2\frac{C_{yx_4}}{Cx_4x_4}=2.4503609>(1+g)$	The ratio estimator
Primary and secondary enrolment (x_5)	$2\frac{C_{yx_5}}{C_{x_5x_5}}=3.78902>(1+g)$	The ratio estimator
Life expectancy at birth (x_6)	$2\frac{C_{yx_6}}{C_{x_6x_6}}=20.673222>(1+g)$	The ratio estimator
Per capita income (x_8)	$2\frac{C_{yx_8}}{C_{x_8x_8}}=1.2941599<2g$	The proposed estimator
Percent urban (x_{10})	$\frac{C_{yx_{10}}}{C_{x_{10}x10}}=0.467738<2g$	The proposed estimator

* Criteria derived from the comparison of the mean square errors of the proposed estimator with other estimators.

Table (5) Relative Efficiency of the Multivariate Product type of Estimator

The variable	The estimator	The mean square error	The relative efficiency
Number of women per family planning center (x_1)	44.301375	161.87808	129.87713
Percentage contribution of the family of the future in the distribution of contraceptives (x_3)	46.301025	180.58414	116.42363
Literacy rate for population 10 years and more (x_4)	46.388216	87.16753	241.19989
Primary and secondary school enrolment (x_5)	45.954854	110.522222	190.22655
Life expectancy at birth (x_6)	45.635646	49.149856	427.75834
Per capita income (x_8)	46.52271	165.02871	127.3976
Percent urban (x_{10})	45.761959	168.84238	124.52005
X_6 , X_4	46.428149	69.662972	301.79966
The mean per element	44.05	210.24261	100

Conclusion:

We can get an estimate of the contraceptive prevalence rate for any governorate depending on the data of the sample of the governorates only.

Table (5) shows that life expectancy at birth (x_6) is the best auxiliary variables can be used to estimate the contraceptive prevalence rate among the variables which have positive correlation with the contraceptive prevalence rate.

Table (5) also shows that literacy rate for population 10 years and over (x_4) is the second best auxiliary variable can be used to estimate the contraceptive prevalence rate. Table (5) also show that using the multivariate version (x_6 , x_4) improved the precession than using one auxiliary variable (x_4).

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