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Recent Trends in Fertility Analysis

By
Youssef Mostafa Mahgoub

I Introduction:

Fertility levels are being measured by several indicators. Some of these indicators are : Total fertility rate (TFR) , General fertility rate (GFR), Crude birth rate (CBR), Age at first marriage, Age at first birth, Age at last birth, Birth intervals, Number of children ever born, Number of living children, Parity progression ratios, and Net reproduction rate. Some of these indicators are discrete variables such as Number of children ever born and Number of living children. Others are theoretically continuous such as the rates and those indicators related to age (age at first marriage, ...). Of course, these indicators can be partitioned to create ordinal variables. Here, we will introduce brief definitions to some of these indicators.

II Fertility indicator definitions.

a- Crude birth rate is the most widely used indicator for fertility all over the world. It is defined as number of births for every 1000 of the population at the middle of the year in a given year.

b- General fertility rate (GFR) is defined as number of births for every 1000 woman aged 15-49 at the middle of the year in a given year.

c- Total fertility rate (TFR) is the average number of births per woman all over her reproductive lifetime. It is a period measure based on the reproductive performance of hypothetical cohort of women who experience the observed age-specific fertility rates over their reproductive lifetime in a particular population at a particular time. It is obtained by summing age-specific fertility rates for single years of age over the childbearing span.

d- An age-specific birth rate is defined as the number of births to persons of a given age group per 1000 woman in that age group.

e- Parity refers to the number of children a woman has already borne alive.

f- Parity progression ratios (PPRs) indicate the proportion of women at each parity who proceed to have another birth.

g- The gross reproduction rate (GRR) is a special case of the total fertility rate. Whereas the total fertility rate measures the total number of children a cohort of women will have , the gross reproduction rate measures the number of daughters it will have.

h- The net reproduction rate (NRR) is a measure of the number of daughters that a cohort of a newborn girl babies will bear during their lifetime assuming a fixed schedule of age- specific fertility rates and a fixed set of mortality rates.

III Methodology for fertility estimation.

Fertility estimations are carried out for the purpose of fertility analyses. Therefore, we will briefly discuss some direct and indirect methods for fertility estimation. Hobcraft (1980) estimated current fertility levels by direct estimates such as fertility rates using data concerning the date of the last live birth extracted from household questionnaire. The author estimated current parity from data about total numbers of children ever borne. He also used marriage duration and birth order to get fertility estimates. As for estimation of past trend in fertility , Hobcraft (1980) used changing educational composition, own-children and maternity histories for that purpose. United Nations Manual IV discussed several methods of estimating basic fertility measures from incomplete data.

III -1 Fertility models:

III -1-1 Coale and Trussell Model:

Coale and Trussell (1974) proposed a model that, by generalizing the pattern of natural fertility, was able to represent the fertility experience of populations where voluntary fertility control was exercised. This model is based on the following assumption: natural fertility either follows marital fertility (if deliberate birth control is not practiced); or it departs from natural fertility in a way that increases with age according to a typical pattern. Therefore, if one denotes by $\phi(x)$ marital fertility at age x and by $h(x)$ natural fertility at the same age, in a population where fertility is controlled voluntarily,

$$\phi(x) = Mh(x)\,\delta(x) \qquad (*)$$

where M is a parameter indicating the level of natural fertility that the population would experience in the absence of all voluntary control and $\delta(x)$ is a function of age indicating the typical pattern of departure from natural fertility when voluntary control is exercised.

III -1-2 Brass relational Gombertez fertility models:

Denoting by $F(x)$ cumulated fertility up to age x and by TF total fertility , the ratio $F(x)/TF$, the proportion of total fertility experienced up to age x , is assumed to follow a Gompertz distribution function, whose form is:

$$F(x)/TF = \exp(A \exp(Bx)) \qquad (1)$$

where A and B are constants ; $A<0$.

This expression can be transformed to:

$$\ln(-\ln(F(x)/TF)) = \ln(-A) + Bx \qquad (2)$$

The ratio $F(x)/TF$ is < 1, thus $\ln(F(x)/TF)$ is negative and the logarithm of a negative number is not defined. Thus, the minus sign must be introduced as in equation (2).

IV Methodologies for fertility analysis.

In the following we will discuss some of the methodologies used by demographers in the literature. We will go quickly through the basic methodology equations and the basic assumptions as will as reporting some of the studies which utilized that methodology.

These methodologies are:

1- Correlation and regression analysis.
2- Time series analysis.
3- Path analysis.

4- Standardization and Multiple Classification Analysis.
5- Hazard models.
6- Experimental Design.
7- Simulation Models.
8- Categorical data analysis.

IV-1 Correlation and Regression analysis.

Draper and Smith (1981) defined multiple regression equation as:

$$Y = X\beta + \varepsilon$$

where Y is an (n x1) vector of observations,
X is an (n x p) matrix of known form,
β is a (p x 1) vector of parameters,
ε is an (n x 1) vector of errors.
and where E(ε) = 0 , V(ε) = Iσ2, so the elements of ε are uncorrelated .

The linear logistic model suggested by Cox is regressing the probability of success on independent variables as follows:

$$p_i = \frac{\exp(\sum_{j=1}^{p} b_j x_{ij})}{1+\exp(\sum_{j=1}^{p} b_j x_{ij})}$$

and

$$1 - p_i = \frac{1}{1+\exp(\sum_{j=1}^{p} b_j x_{ij})}$$

Let $\lambda_i = \log_e \frac{p_i}{1-p_i} = \sum_{j=1}^{p} b_j x_{ij}$ (*)

where bj are unknown coefficients.

λi is called the logistic transform of Pi and Equation (*) is a linear logistic model. Another name for λi is log odds (Lee E T (1980)).
Varea C, et all (2000) used correlation to show the association between some fertility measures and ecological determinants. The results show an association at the individual level between age at menarche and particular characteristics of ovarian function (fetal loss) and age at menopause. Di Giulia P. et all (1999) used correlation procedure to study the association between age at first birth and some socioeconomic variables. Tawiah EO (1999) used logistic regression to study the effects of level of education, region of residence, experience of child loss and religion on male fertility preference. Togunde OR (1999) used logistic regression models to estimate employment effects at specific parities during a 5-year period for Nigerian women. Andersson G (1999) used regression procedure to study the impact of labor-force participation and other factors such as the level of women's earning on the levels of childbearing in Sweden.

IV-2 **Time series analysis**

This is a powerful statistical technique if there is a reliable time series data over a long time period. Some scholars used time series analysis to model and predict some fertility measures whenever time series data is available. This methodology attempts to measure the impact of family planning programs on fertility by comparing indicators of actual fertility (observed fertility) over a specific period of time to projected fertility data for the same time period (Abd El-Fattah(2000)).

Keil T J and Andreescu V (1999) used time series analysis to study the Romanian national birth rate between 1967 and 1989. Kohler (2000) used the autoregressive process to determine the net reproduction rate (NRR) using AR (1) model. The author used time series data for NRR from 1930 - 1994.

Hussein M A (1993) used time series technique to analyze crude birth rate till year 2010. The author suggested that ARIMA (1,1,0) is the model to be considered for the natural logarithms of the data. We will introduce this model as follows:

Model:

$$(\Delta \ln z_t - \mu) - \phi(\Delta \ln z_{t-1} - \mu) = a_t$$

which can be defined as:

$$(1 - \phi\beta)(\Delta \ln z_t - \mu) = a_t$$

z_t is the observed time series at time t.

Δ is the difference operator.

μ is $E(z_t)$

ϕ is the autoregressive parameter such as $|\phi| < 1$.

β is backward shift operator that shifts time one step back.

a_t is the error at time t.

IV -3 Path Analysis.

The path analysis model is used in order to determine the direct and indirect paths through which the independent variables affect the dependent variable. In path analysis , the variables are arranged in order so that each variable depends on the previous variables . Each equation represents a causal link rather than a mere empirical association.

Boadu (2001) used path analysis to study the direct and indirect effects of respondent's education described as a function of four other exogenous background variables on number of children ever born (CEB).

IV -4 Standardization and Multiple Classification Analysis:

Multiple Classification Analysis (MCA) is considered to be as an extension of standardization because the observed means can be adjusted by fitting additive models.

MCA model can be expressed by the following equation:

$$Y_{ijk} = Y + a_i + b_j + \ldots\ldots + e_{ijk}$$

where Yijk = score of a particular individual who falls into i-th category of predictor A, J-th category of predictor B, etc,
Y = grand mean of Y,
ai = Added effect of i-th category of predictor A (= difference between Y and the mean of i-th category of predictor A),
bj = Added effect of j-th category of predictor B (= difference between Y and the mean of j-th category of predictor B),

The coefficients for a certain predictor estimated by solving the normal equation system which reveal the closeness of the relationship between the predictors and the dependent variable.
Wigger BU(1999) used MCA to study the impact of level of education, region of residence, experience of child loss and religion on male fertility preference as measured by mean ideal number of children.
El-Deeb B M(1990) used standardization technique to estimate the effect of changes in marital status, marital fertility and age structure on Egyptian fertility 1960-1986. Khalifa M and Farahat A(1993) used MCA and regression analyses to determine the effect of socioeconomic variables on the quantity and speed of progression to the next parity .Farahat A , Abd El-fattah M and Mahgoub Y (1992) used MCA to study socio - economic determinants of achieved fertility in Egypt.

IV -5 **Hazard Models:**

Nair (1996) used Cox's proportional hazard model to estimate the effects of socio-economic , demographic and proximate variables on birth interval in Kerala (India). Shoeib (1998) used proportional hazard model to study the influence of socio-demographic variables on the onset of childbearing and the pace of subsequent births.

IV- 6 **Experimental Design:**

This technique compares two or more groups, one of these groups is the "control" group and the others are "experimental" or study groups. Under the assumption of normality, t-test (paired or for two independent samples) and one way analysis of variance (in case of independent samples) are used. Demographers have used this technique as a method of measuring the impact of family planning programs on fertility (Abd El-Fattah 2000). Levine PB; Staiger D; Kane TJ; Zimmerman DJ (1999) used quasi-experimental method to estimate 1) the effects of legalization of abortion by a group of individual states before Roe v Wade decision and 2) the effects of the Roe decision. Variation in the timing of state level abortion legalization across states was used to compare birth data between (pseudo) control and treatment groups.

To find a relation between changes in fertility during time and changes in infant or child mortality some time before say t-k Palloni and Rahalimananal (1999) introduced the following model :

$$\delta F(i,t) = \alpha + \beta\, \delta Q(i, t\text{-}k) + \gamma\, X(i, t\text{-}k) + Vi + \varepsilon it$$

where δ F(i,t) is the the relative or proportionate change in the total fertility rate observed in country i during interval (t- k, t) , δQ(i, t-k) is the relative or proportionate change in infant mortality rate observed in country i during the the time interval (t - 2k, t - k);

X(i , t -k) is a measure of socioeconomic conditions at time t -k, reflecting social and economic factors affecting both F and Q at the beginning of each decade; Vi is a fixed effect for country i; εit is a homoscedastic normally distrbuted error term , uncorrelated with Q(i,t) , X(i,t), and Vi; and β is the effect associated with infant mortality.

IV -7 **Simulation Models;**

Demographers have used simulation models to estimate fertility levels. Palloni and Rafalimanana (1999) used simple Monte Carlo simulation of the survival status of a child that opens an interval. The authors estimated total fertility rates for three different scenarios of child mortality. Simulation models are also used to study averted births by the use of birth control methods (Abd El- Fattah).

IV - 8 **Categorical Data:**

Since some basic fertility indicators are rates and proportions, a frequency table can be generated to study fertility association with some socioeconomic variables (nominal or ordinal or partitioned continuous variables). Log linear models are used to analyze data in frequency tables. In a fourfold table Fleiss (1980) introduced different methods for data analysis, such as, relative risk, attributable risk and odd ratios and showed how to make statistical inferences about them. As a matter of fact, odds ratios are the most widely used method in fertility analysis with respect to frequency tables.

IV - 8-1 **Odds ratios:**

Razzaque A (1999) used odds ratios to study the preference for children and subsequent fertility in Matiab, Bangladesh. Mahgoub Y (1989) used global odds ratios to study the relationship between women's education and number of their living children.

Here, global odds ratios and their interpretations are introduced below.

$$\Psi_{ij} = \frac{\{\sum_{a \le i} \sum_{b \le j} \Pi_{ab}\} * \{\sum_{a > i} \sum_{b > j} \Pi_{ab}\}}{\{\sum_{a > i} \sum_{b \le j} \Pi_{ab}\} * \{\sum_{a \le i} \sum_{b > j} \Pi_{ab}\}}$$

for i=1,21,......... ,r-1 (rows), j=1,2,......., c-1 (columns), where Π_{ij} denotes the population proportion in the cell (i,j) and

$$\sum_{i=1}^{r} \sum_{j=1}^{c} \Pi_{ij} = 1$$

An example for global odds ratio and its interpretation is given using the data and the results shown in Table (1). Each global odds ratio can be expressed as a ratio of odds of cumulative events as:

$$\Psi_{ij} = \frac{odds(c_1 \subseteq i / c_2 \subseteq j)}{odds(c_1 \subseteq i / c_2 \supset j)} = \frac{odds(c_2 \subseteq j / c_1 \subseteq i)}{odds(c_2 \subseteq j / c_1 \supset i)}$$

Odds for an event E are defined as:

$$\frac{p(E)}{1-p(E)}$$

Semenya and Kock (1980) discussed some global odds ratio models and showed how to analyze them with weighted least squares (WLS) approach. Mahgoub (1989) showed how to analyze global odds ratio models using maximum likelihood approach. The class of global odds ratio models as well as the formulation of the problem are described as
follows:

Assuming a simple multinomial sampling in which the constraints by sampling design is $\sum_{i=1}^{r}\sum_{j=1}^{c}\Pi_{ij}=1$. let nij be the observed frequencies at cell (i,j) , I= 1,2,.......,r , j=1,2,......c. The kernel of the log-likelihood function is

$$\sum_{i=1}^{r}\sum_{j=1}^{c} n_{ij}\ln(\pi)_{ij} \quad \text{or equivalently} \quad \sum_{i=1}^{r}\sum_{j=1}^{c} n_{ij}\ln(m)_{ij}$$

where mij are the MLEs of the cell frequencies and $\sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}=n$,the total sample size. Therefore, Π, of Π will be the solution of the following problem:

Maximize $\sum_{i=1}^{r}\sum_{j=1}^{c} n_{ij}\ln(\pi)_{ij}$

subject to:

1) A1 ln A2 Π = Xβ,

2) $\sum_{i=1}^{r}\sum_{j=1}^{c}\Pi_{ij}=1$, or $\sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}=n$

Nelder and Mead (1965) have developed a simple procedure for solving minimization (maximization) problems using the simplex procedure . This procedure was utilized to get MLEs estimates of Π and β.

The following is an example for global odds ratio interpretation. The global odds ratio at Secondary / Higher 4-5/6+ cutpoint (Table (1)) means the odds on having at most five living children given that the level of education is "Higher" is 2.64 times the odds on having at least six living children given that the level of education is "Higher".It also means :for those women whose level of education is "Higher" the odds on having at most five children is 2.64 times the odds on having at least six children. This means that women with "Higher" education are much less probable to have six or more children.

Table (1): Education and No. of Living Children (DHS, 1988)
Rural Upper Egypt

Level of Education	Number of Living Children						
	0-1		2-3		4-5		6+
Higher	6		3		1		1
		3.24		3.80		2.64	
Secondary	32		22		12		1
		2.66		3.68		10.41	
Primary	132		143		133		95
		1.16		1.24		1.44	
No Education	398		401		374		338

*: Global Odds Ratios

V Limitations of the Methodologies Used in Fertility Analysis.

It should be noted that every methodology has its own limitations which the analyst should be aware of. Here, we will discuss briefly the basic limitations for some of these methodologies.

A) Regression analysis requires that the dependent variable must be normally distributed. Unless the sample size is relatively large, the normal assumption for some fertility measures (for instance, number of children ever born) is hardly accepted. However, large scale survey data tend to contain measurement errors in all the variables included in the equation (Ogawa 1980). It is also well known that multicollinearity is a serious problem in case of socioeconomic explanatory variables.

B) Path analysis considers only causal relationships not merely empirical association. Several important socioeconomic variables are not causally related, but rather have a strong association, for instance, husband's education and wife's education.

C) As for time series technique, it is a powerful statistical technique but it requires a reliable relatively long time series data which may not be available in many countries.

D) Standardization should never substitute for a comparison of the specific rate themselves (Fleiss 1980). He added that "It is these that characterize the experience (morbidity, mortality, or whatever the rate refers to) of the population being studied. One criticism of the adjustment of rates is that if the specific rates vary in different ways across the various strata, then no single method of standardization will indicate that these differences exist. Standardization will, on the contrary, tend to mask these differences (Fleiss 1980). El-Badry pointed out that in Ceylon, India, and Pakistan mortality among males occurs at a lower rate than among females in many age categories. Thus, single summary indices for males and females might mask this phenomenon and thus fail to reveal data suggestive of further research. Multiple classification technique requires a substantial number of observations for obtaining reliable estimates of means. Moreover, inter correlation among predictors causes serious difficulties in computing the values for their coefficients. Unless predictors are statistically independent, the total of sums of squares for gross effects would be either more or less than, but not equal to, the sum of squares for the additive model (Ogawa 1980).

E) Hazard models assume that fertility is uncontrolled which is not valid in many populations now. We also believe that such models should be conditional on parity.

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