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Time Series Analysis for Forecasting Mortality and Fertility In Egypt till 2020 Using Intervention Models

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Summary

The Egyptian population is making a noticeable Progress concerning fertility and mortality. This Progress has started as early as the Second World War. In this study, some fertility and mortality indicators were selected. Using time series analysis, two types of models considered in the analysis. The univariate and the intervention models are considered. The best model for each indicator was estimated and checked. These models were used to get predictions till 2020. The steps as well as the method for selecting the best model are presented. The predicted values for the fertility and mortality measures are given.

1-Introduction:-

In this paper, the author used time series analysis to study some fertility and mortality measures to get their projections till 2020. Hussein (1993) used time series technique to analysis crude birth rate and crude death rate and estimated their projections till 2010. Hussein and Mahgoub (2000) used time series technique to analyze some mortality indicators. The best model for each indicator was estimated and checked. These models were used to get predictions till 2010. In this paper one fertility measure will be used for fertility prediction up to 2020. Also, in this paper three mortality measures will be used for mortality predictions up to 2020. These mortality measures are crude death rate (CDR), infant mortality rate (IMR) and life expectancy at birth (LEB). For crude birth rate and crude death rate the data are available from the beginning of the past century till year 2003. Data for infant mortality rate started from 1947. Life expectancy at birth was available only from 1970. Time series analysis is a powerful statistical technique if there is a reliable time series data over a long time period. This methodology attempts to measure the impact of family planning and health care programs on fertility and mortality rates by comparing the actual indicators of fertility and mortality with the projected ones for the same period of time. Time series models can be classified according to the number of variable included in the model into two types of models univariate and multivariate time series models (Walti Vand 1992). Univariate models consider one variable only in the analysis. In this type of models we assume that the factors determine this variable will not be changed or we are not expecting a notable change to be considered in the model. The other type is the multiple time series model or transfer function model which contain one or more independent variables as explanatory variables. The class of ARIMA models applied to the estimated values from the regression model. The intervention model is a special class of the multiple time series model. In this model the number of the independent variables is not important. The intervention model should contain at least one of the independent variables, can be changed by new law or by new policy. In this paper, the two types of models are used to get the predicted values (Hussein, 1993).

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The second type of the analysis is more powerful than the first type for two reasons: The second type of time series analysis considers the dynamic relationship between the variable of interest and other factors in the population. In this type of analysis we can assume a change in the independent variable and than calculate the predicted values for the variable of interest if the change in the independent variable take place. In this paper four measures for the whole economy are used for the multiple time series. These measures are: the number of employed persons, domestic product, investment and salaries.

2- Objectives:

The main objectives of this research are:

1. Using the available time serried data for fertility and mortality indicators and some economic indicators for the whole economy of Egypt to get the predicted values for each of the mortality and fertility indicators till 2020.
2. Using the intervention models to get the predicted values till 2020. These fitted values and models can be used as reliable tools for policy implications.
3. Comparing the predicted values using the univariate analysis with that using the multivariate analysis. Comparing the results of the multivariate analysis with the results of other authors for the same period of time.

3- Data Sources:

The time series data is shown in table (1) for the crude birth rate (CBR) and crude death rate (CDR) from 1900-2003. The long time series data for both crude birth rate and crude death rate shown in table (1) have been collected from different sources. The main sources were vital statistics and statistical year books issued by Central Agency for Public Mobilization and Statistics (CAPMS).A time series data is shown in table (2) for infant mortality from 1947 till 2000. Life expectancy at birth was available only from 1975. Life expectancy at birth data were collected mainly from national human development reports published by the national planning institute as well as the statistical year book published by CAPMAS.

Another time series data is shown in table (3) for some economic indicators for Egypt from 1960 to 2000. The main source of these data is the reviewable document for the most important variables of the national economy from 1960 to 2000. This document is issued by the Ministry of Planning. These data are collected for the purpose of its use in the multivariate analysis.

Table (1) CBR and CDR in Egypt
From 1900-2003

Year	CBR	CDR	Year	CBR	CDR
1900	43.1	32.0	1927	42.7	24.5
1901	41.7	22.4	1928	43.6	26.3
1902	43.5	27.7	1929	44.2	27.6
1903	43.7	23.6	1930	45.4	24.9
1904	43.8	27.5	1931	44.5	26.6
1905	44.5	25.5	1932	42.5	28.5
1906	46.3	25.1	1933	43.8	27.5
1907	45.8	28.3	1934	42.8	27.8
1908	47.5	26.3	1935	41.3	26.4
1909	44.4	27.9	1936	44.2	28.8
1910	45.8	37.6	1937	43.4	27.1
1911	45.4	29.0	1938	43.2	26.3
1912	44.8	25.9	1939	42.0	25.9
1913	44.1	26.8	1940	41.3	28.3
1914	44.7	28.5	1941	40.4	27.9
1915	43.9	29.4	1942	37.6	30.5
1916	42.1	31.3	1943	38.7	30.4
1917	42.2	30.8	1944	39.8	28.6
1918	39.0	29.7	1945	42.7	30.2
1919	38.3	29.8	1946	41.2	27.5
1920	42.8	28.4	1947	43.7	21.4
1921	42.3	25.3	1948	42.6	20.4
1922	43.2	25.2	1949	41.6	20.5
1923	43.0	25.7	1950	44.2	19.0
1924	43.3	24.6	1951	44.6	19.2
1925	42.5	26.0	1952	45.2	17.8
1926	43.3	26.3	1953	42.6	19.6

**Table (1): (continued) CBR and CDR in Egypt
From 1900-2003**

Year	CBR	CDR	Year	CBR	CDR
1954	42.6	17.9	1982	36.2	10.0
1955	40.3	17.6	1983	36.8	9.7
1956	40.7	16.4	1984	38.6	9.5
1957	38.0	17.8	1985	39.8	9.4
1958	41.1	16.6	1986	38.7	9.2
1959	42.8	16.3	1987	37.4	9.1
1960	43.1	16.9	1988	36.6	8.1
1961	43.9	15.8	1989	33.3	8.1
1962	41.3	17.9	1990	32.2	7.5
1963	42.8	15.4	1991	30.8	7.5
1964	42.0	15.7	1992	27.7	9.0
1965	41.4	14.0	1993	29.0	8.0
1966	41.0	16.8	1994	28.6	6.4
1967	39.2	14.2	1995	27.9	6.7
1968	38.2	16.1	1996	28.3	6.5
1969	37.0	14.5	1997	27.5	6.5
1970	35.1	15.1	1998	27.5	6.5
1971	35.2	13.2	1999	27.0	6.5
1972	34.5	14.5	2000	27.4	6.3
1973	35.9	13.1	2001	26.7	6.3
1974	35.8	12.7	2002	26.3	6.4
1975	36.2	12.2	2003	26.1	6.5
1976	36.6	11.8			
1977	37.5	11.8			
1978	37.4	10.5			
1979	40.2	10.9			
1980	37.5	10.0			
1981	36.8	10.0			

Table (2): IMR and LEB in Egypt
From 1947-2000

Year	IMR	LEB	Year	IMR	LEB
1947	127		1974	101	
1948	139		1975	89	55
1949	135		1976	87	55
1950	130		1977	85	
1951	129		1978	74	
1952	127		1979	76	
1953	146		1980	71	57
1954	138		1981	71	57
1955	136		1982	71	57
1956	124		1983	65	58
1957	130		1984	62	58
1958	112		1985	49	57
1959	109		1986	47	61
1960	109		1987	49	61
1961	108		1988	43	63
1962	134		1989	40	60
1963	118		1990	38	60
1964	117		1991	36	61
1965	113		1992	36	62
1966	127		1993	32	64
1967	116		1994	31	64
1968	131		1995	30	64
1969	119		1996	29	67
1970	116	52	1997	30	68
1971	103		1998	27	68
1972	116		1999	26	69
1973	98		2000	25	65

Table (3): Data for Economic Variables in Egypt From1960 to 2000

Year	Employment**	Domestic product*	Investment*	salary*
1960	6235.0	1324.2	171.5	559.2
1961	6749.1	1388.7	225.6	612.0
1962	6897.4	1441.6	251.6	668.9
1963	7117.1	1583.5	299.6	771.4
1964	7344.8	1730.7	372.4	863.2
1965	7574.0	1823.8	364.7	956.7
1966	7807.8	1909.0	383.8	1047.4
1967	7835.4	1917.8	365.8	1075.5
1968	8021.0	1859.4	297.8	1099.0
1969	8147.1	1971.8	343.2	1178.5
1970	8383.4	2806.0	355.5	1264.3
1971	8506.0	2939.9	361.5	1347.4
1972	8671.5	3066.1	369.5	1432.9
1973	8888.3	3147.1	466.7	1590.8
1974	9041.7	3415.2	687.8	1771.8
1975	9433.3	5061.3	1282.3	2140.6
1976	9628.2	5526.6	1471.1	2635.4
1977	9885.5	5906.0	1873.3	2946.7
1978	10216.3	6538.8	2684.8	3427.2
1979	10554.0	7119.1	3763.0	3987.3
1980				
1981	11439.1	11439.1	5334.4	5930.4
1982	10522.0	10522.0	6286.5	9193.1
1983	10795.0	10795.0	8290.3	10507.7
1984	11072.0	11072.0	9150.9	11844.3
1985	11367.0	11367.0	10628.9	13384.8
1986	11669.0	11669.0	13014.4	1488.5
1987	11998.0	11998.0	14593.5	16186.4

** Thousand Employees
* Million pounds

Table (3): Data for Some Economic Variables in Egypt From1960 to 2000

Year	Employment**	Domestic product*	Investment*	salary*
1988	12334.0	51840.0	21798.2	19379.3
1989	12685.0	54264.0	24148.9	22067.7
1990	13032.0	56845.0	26181.4	25578.5
1991	13376.0	58923.0	25478.0	29705.4
1992	13742.0	131057.0	27504.5	33963.7
1993	14011.0	134335.0	31644.0	38583.9
1994	14436.0	139622.0	33452.0	44547.6
1995	14879.0	146131.0	39412.0	51900.3
1996	15340.0	153369.0	44106.0	60042.3
1997	15825.0	239500.0	55280.0	69893.3
1998	16344.0	253090.0	62010.0	77003.6
1999	16874.0	299597.2	68587.0	85666.0
2000	17434.0	285847.0	73106.0	95622.5

The following sections illustrate the univariate and the multivariate analysis for each demographic indicator.

4-Crude Birth Rate (CBR)

Introduction:

Fertility as measured by crude birth rate (CBR) has declined substantially from 43.1 per thousand in 1900 to 26.1 per thousand in 2003 but it is still much higher than what is hoped for. Table (1) shows that crude birth rate goes up wards and downwards in the short run, but it obviously decreases in the long run. This substantial decrease is due to socioeconomic development that took place in the Egyptian society during this century as well as family planning programs. However, changes in age structure of females in the reproductive age group (15-49) eliminated a great part of the negative effect of family planning programs (Hussein, 1993).

Step 1: Model Identification

The first step in model Identification as shown in figure (1) is to plot the series of crude birth rate. The plot of the crude birth rate goes up and down. The plot shows that the levels of the series change with time, which means that the series is non-stationary (Abraham & Ledoltter 1983). To verify this pattern we inspected the autocorrelation function (ACF). The ACF plot for the time series started out with large positive value which died out very slowly as shown in figure (5). This pattern confirms that the series is not stationary, and that we must take differences when analyzing them. The plot of the series indicated that the variance changes with time; the natural logarithmic transformation (base e) is used to stabilize the variance. Consequently, we considered the natural logarithmic transformation of the series in the analysis. The second step in model identification is to plot the autocorrelation as well as the partial autocorrelation function (PACF). The plot of ACF shows exponential decay. Moreover the plot of PACF showed cut off after lag 1. So, ARIMA (1,0,0) and

ARIMA (1,1,0) are highly suggested. The second model is proven to be more efficient without constant. The second model is significant.
Model 1 : Crude Birth Rate

$$Z_t - \phi Z_{t-1} = a_t$$

This can be simplified to

$$(1- \phi B) Z_t = a_t$$

Where Z_t is the observed time series
a_t is the error at time t
ϕ is the autoregressive parameter such that $|\phi| < 1$
B is backward shift operator that shifts time one step back.

Step 2: Model Estimation:

Table (4) shows the final parameters estimates for model 1. This table shows that the autoregressive parameter ϕ is highly significant (P-value = .000). The mean of the series μ which is the constant in the model is not significant, so it is not considered in the model. Therefore, ARIMA (1, 0, 0) without constant seems to be a good model for CBR data.

Table (4): Final Parameters Estimates, Model (1)
Crude Birth Rate (Univariate Analysis)

Number of residuals 104				
Standard error		184, 74504		
		Analysis of variance		
	DF	Adj. Sum of squares	Residual variance	
Residuals	103	.17403705	.00156690	
		Variables in the Model		
	B	SEB	T-RATIO	APPROX. PROB.
ARI	0 .99980434	.00022002	4.5440812	.00000000

Analysis will be applied to the natural logarithm of the data.

R^2=. 86

5- Crude Death Rate (CDR)

Introduction:

Mortality as measured by crude death rate (CDR) has also declined – substantially from 24.5 per thousand in 1900 to 6.5 per thousand in 2003. This decline is so clear after Second World War because of the improvement in the overall health status that occurred in the Egyptian society. It goes without saying that the prediction of the future crude death rate is vital.

Step 1: Model Identification:

To identify the best model, the time series of CDR should be plotted against time to recognize its change pattern. Figure (2) shows that CDR changes with time which means that the series is non-stationary .Figure (2) shows that CDR had almost the same level from the beginning of the twentieth century till the middle of 1940's. After the Second World War crude death rate started to decline gradually until it reached 6.5 in year 2003. Figure (2) shows also that the variance changes with time. The natural logarithmic transformation proved to stabilize the variance. Thus, we will use the successive differences and natural logarithmic form for model selection. Figure (6) shows the autocorrelations and the partial autocorrelation of the natural logarithm of CDR. For crude death rate, the plots of ACF indicated exponential decay. More over the plot of PACF showed cut off after lag 1. Therefore ARIMA (1,1,0) for the natural logarithmic transformation is highly suggested. The model for crude death rate can be illustrated as follows.

Model 2: Crude Death Rate

$$\Delta \ln z_t - \mu - \phi(\Delta \ln z_{t-1} - \mu) = a_t$$

Which can be simplified to :

$$(1- \phi B)(\Delta \ln z_{t-1} - \mu) = a_t$$

Where Z_t is the observed time series at time t

Δ is the difference operator

μ is E (Zt)

at is the error at time t

ϕ is the autoregressive parameter such that $|\phi| < 1$

B is backward shift operator that shifts time one step back.

Step 2: Model Estimation:

Table (5) shows the final parameters estimates for model 2. This table shows that the autoregressive parameter ϕ is highly significant (p-value< (0000001) The mean of the series u which is the constant in the model is also significant (p-value=.016033) Therefore, ARIMA (1,1,0) seems to be a good model for CDR data.

Table (5): Final parameters estimates, model (2) crude death rate (univariate analysis)

Number of residuals	103			
Standard error	.08754013			
Log likelihood	105.60576			
Analysis of variance				
	DF	adj .sum of squares		Residual variance
Residuals	101	.77577386		.00766327
Variables in the model				
	B	SEB	T-RATIO	APPROX. PROB.
ARI	-.459378	.08807609	-5.215697	.00000098
Constant	-.014521	.0059286	-2.44934	.01603311

Analysis will be applied to the natural logarithm of the data.

$$R^2 = .99$$

6- Infant Mortality Rate

Introduction:

Social studies experts have always considered infant mortality rate as an important measure for national levels of modernization. Also, levels and trends of infant mortality rate (IMR) have generally declined from almost 101 per thousand in 1947 to 25 per thousands in 2003.

Step 1: Model Identification:

To identify the best model, the time series of IMR is plotted against time to recognize its change pattern. Figure (3) shows that IMR decreases almost linearly with time. Figure (7) shows the autocorrelations and the partial autocorrelation of the natural logarithm of IMR. For IMR, the plots of ACF indicated exponential decay. Moreover the plot of PACF ' showed cut off after lag 1. Therefore, ARIMA (1,1,0) for the natural logarithmic transformation is highly suggested. The model for infant mortality rate can be illustrated as follows:

Model 3: Infant Mortality Rate

$$(\Delta \ln Z_t - \mu) - \phi(\Delta \ln Z_{t-1} - \mu) = a_t$$

This can be simplified to

$$(1-\phi B)(\Delta \ln Z_{t-1} - \mu) = a_t$$

Where Z_t is the observed time series at time t

Δ is the difference operator

μ is E (Zt)

at is the error at time t

ϕ is the autoregressive parameter such that $|\phi|<1$

B is back word shift operator that shifts time one step back.

Step 2: Model Estimation:

Table (6) shows the final parameters estimates for model 3. This table shows that the autoregressive parameter ϕ is highly significant (p-value<.007) .The mean of the series μ which is the constant in the model is also significant (p-value<.0003) Therefore, ARIMA (1,1,0) for the natural logarithmic transformation seems to be a good model for IMR data.

Table (6): Final Parameters Estimates, Model (3) Infant Mortality Rate (univariate analysis)

Number of residuals	53			
Standard error	.0777229			
Log likelihood	61.120027			
		Analysis of variance		
	DF	adj. Sum of squares	Residual variance	
Residuals	51	.30893284	.00604085	
		Variables in the model		
	B	SEB	T-RATIO	. Approx. prob.
AR1	- .36849938	.12960093	- 2.843339	.00640723
Constant	- .03124317	.00784123	- 3.984472	.00021559

(Analysis will be applied to the natural logarithm of the data).

$R^2 = .98$

7- Life Expectancy at Birth (LEB)

Introduction:

Life expectancy at birth (LEB) in Egypt has been increasing from 57 years in 1953 to 65 years in 2000. This means that the Egyptian citizen has gained on the average twelve more years to his life span during two decades which is substantial increase

Step 1: Model Identification:

To identify the best model, the time series of LEB was plotted against time to recognize its change pattern. Figure (4) shows that LEB increases with time. The sample autocorrelations and partial autocorrelations are shown in Figure (8). This figure indicates a cut off in the ACF and a rough exponential

decay with oscillatory pattern in the PACF. Thus we consider ARIMA (0,0,1) model.
The model for life expectancy at birth can be illustrated as follows:

Model 4: Life expectancy at birth

$$(\ln Z_t - \mu) = (1 - \theta B)\, a_t$$

Where θ is the moving average parameter such that $|\phi| < 1$

Z_t is the observed time series at time t
μ Is E (Zt)
at is the error at time t
B is backword shift operator that shifts time one step back.

Step 2: Model Estimation:

Table (7): shows the final parameters estimates for model 4. This table shows that the moving average parameters θ is highly significant (p-value<.001). The mean of the series μ which is the constant in the model is also significant (p-value<.001) therefore ARIMA (0,0,1) seems to be a good model for LEB.

Table (7): Final parameters estimates, model (4) Life Expectancy at Birth (univariate analysis)

Number of residuals	24			
Standard error	.05297734			
Log likelihood	36.456993			
		Analysis of variance		
	DF	adj. Sum of squares	Residual variance	
Residuals	22	.06734639	.0028066	
		Variables in the model		
	θ	SE θ	T-RATIO	Approx. prob.
MA1	-.7783401	.15427631	-5.04510	.00004724
Constant	-4.0989018	.01830176	-223.9622	.0000000

Analysis will be applied to the natural logarithm of the data.

$$R^2 = .96$$

7- Forecasting

Using the univariate analysis, the predicted values with their lower and upper confidence limits are presented in table (8) and table (9) respectively

Table (8): Predicted Values for CBR and CDR Using Univariate Analysis

	CDR				CBR		
Year	LCL	Fit	UCL		LCL	Fit	UCL
2000	5.345	6.364	7.5770		24.94532	26.98259	29.18625
2001	5.255	6.257	7.4500		25.31481	27.38226	29.61855
2002	5.180	6.168	7.3438		24.66821	26.6828	28.86202
2003	5.224	6.221	7.4066		24.29872	26.28318	28.4297
2004	5.307	6.319	7.5232		24.11397	26.08335	28.2135
2005	5.135	6.267	7.6482		23.32781	26.0667	29.12718
2006	4.845	6.159	7.8286		22.73878	26.05008	29.84359
2007	4.652	6.078	7.9412		22.25169	26.03347	30.45799
2008	4.453	5.987	8.0488		21.8296	26.01688	31.00724
2009	4.281	5.902	8.1366		21.4538	26.0029	31.51024
2010	4.118	5.816	8.2153		21.1129	25.98372	31.97821
2011	3.967	5.733	8.2840		20.7997	25.96717	32.41847
2012	**3.825**	**5.650**	**8.3454**		20.5089	25.95063	32.83609
2013	3.692	5.569	8.4001		20.2372	25.93410	33.2347
2014	3.564	5.488	8.4493		19.9814	25.91758	33.6173
2015	3.444	5.409	8.4935		19.7396	25.90108	33.9858
2016	3.331	5.331	8.5335		19.50994	25.88460	34.3421
2017	3.222	5.254	8.5695		19.29110	25.86812	34.6875
2018	3.118	5.179	8.6021		19.0819	25.85167	35.02317
2019	3.018	5.104	8.6316		18.88139	25.83522	35.35008
2020	2.923	5.030	8.6583		18.68875	25.81879	35.66904

Table (9): Predicted Values for IMR and LEB Using Univariate Analysis

	IMR			LEB		
Year	LCL	Fit	UCL	LCL	Fit	UCL
2000	21.579	25.261	29.570	57.174	63.945	71.518
2001	20.761	24.302	28.448	54.583	61.047	68.276
2002	19.499	23.529	28.392	52.175	60.274	69.631
2003	18.200	22.815	28.599	52.175	60.274	69.631
2004	17.120	22.110	28.584	52.175	60.274	69.631
2005	16.123	21.431	28.485	52.175	60.274	69.631
2006	15.218	20.771	28.350	52.175	60.274	69.631
2007	14.382	20.132	23.122	52.175	60.274	69.631
2008	13.606	19.513	27.984	52.175	60.274	69.631
2009	12.884	18.913	27.562	52.175	60.274	69.631
2010	12.209	18.331	27.523	52.175	60.274	69.631
2011	11.576	17.767	27.269	52.175	60.274	69.631
2012	10.982	17.221	27.003	52.175	60.274	69.631
2013	10.423	16.691	26.728	52.175	60.274	69.631
2014	9.896	16.177	26.444	52.175	60.274	69.631
2015	9.399	15.679	26.156	52.175	60.274	69.631
2016	8.931	15.198	25.862	52.175	60.274	69.631
2017	8.487	14.730	25.565	52.175	60.274	69.631
2018	8.068	14.277	25.265	52.175	60.274	69.631
2019	7.671	13.838	24.962	52.175	60.274	69.631
2020	7.295	13.412	24.659	52.175	60.274	69.631

8- Multiple Time Series Model

From section IV to section VIII the author considered the univariate time series analysis for mortality and fertility measures. In the univariate analysis the data for one variable is only considered in the analysis. In this type of models we assume that the factors that determine these variables will not be changed or we are not expecting a notable change to be considered in the model. The other type is the multiple time series model or transfer function model which contains one or more independent variables as explanatory variables. The class of ARIMA models applied to the estimated values from the regression model. The intervention model is a special class of the multiple time series model. In the intervention model the analysis should contain at least one variable that can be changed by new law or by new policy. The following tables illustrate the multivariate analysis. Tables (10) to (13) illustrate the results of the multivariate analysis for CBR, CDR, IMR and LEB. Table (15) and table (16) illustrate the fitting values and the lower confidence interval and the upper confidence interval for CBR, CDR, IMR and LEB respectively. To get the fitting values and the lower confidence interval and the upper confidence interval, we assumed that the economic indicators will be changed as follows:

$$X_t = X_{t-1}\,(1.01)$$

The data for the economic indicators estimated for the years from 2001 to 2020 and illustrated in table (14). The multiple time series model for crude birth rate (CBR), crude death rate (CDR) and infant mortality rate (IMR) can be illustrated as follows:

$$Z_t = \mu + \phi Z_{t-1} + b_1 x_1 + a_t$$

Z_t is the observed time series at time t

μ is E (Z_t)

ϕ is the autoregressive parameter

a_t is the error at time t

b_1 is the regression coefficient

x_1 is the first independent variable

This model is considered as a combined model between the autoregressive parameters and the regression parameters. Tables (10) to (12) illustrate the final parameters estimates for these models. The multiple time series model for life expectancy at birth (LEB) can be illustrated as follows:

$$Z_t = \mu + b_1 x_1 + a_t - \theta a_{t-1}$$

Z_t is the observed time series at time t

μ is E (Z_t)

θ is the moving average parameter such that $|\theta| < 1$

a_t is the error at time t

b_1 is the regression coefficient

x_1 is the first independent variable

Table (13) illustrates the final parameters estimates for this model.

Table (10): Final Parameters Estimates, Model (5) Crude Birth rate (Multivariate analysis)

Number of residuals	43			
Standard error	.03745801			
Log likelihood	80.729832			
	Analysis of variance			
	DF	adj. Sum of squares	Residual variance	
Residuals	40	.05890450	.00140310	
	Variables in the model			
	B	SEB	T-RATIO	Approx. prob.
ARI	.8818846	.07442834	11.848775	.00000000
SAL	-.0000039	.00000090	-4.287246	.00011082
Constant	3.6532790	.04966843	73.553340	.00000000

(Analysis will be applied to the natural logarithm of the data).

$$R^2 = .976$$

Table (11): Final Parameters Estimates, Model (6) Crude Death Rate (Multivariate analysis)

Number of residuals	43			
Standard error	.08487468			
Log likelihood	45.526574			
Analysis of variance				
	DF	adj. Sum of squares	Residual variance	
Residuals	40	.30287266	.00720371	
Variables in the model				
	B	SEB	T-RATIO	APPROX. PROB.
ARI	.8888313	.06312431	14.080650	.00000000
EMP	-.0000422	.00001063	- 3.971505	.00028966
Constant	2.8117128	.15686924	17.923927	.00000000

Analysis will be applied to the natural logarithm of the data.

$R^2 = .976$

Table (12): Final Parameters Estimates, Model (7) Infant Mortality Rate (Multivariate analysis).

Number of residuals	40			
Standard error	.08354167			
Log likelihood	41.899641			
Analysis of variance				
	DF	adj. Sum of squares	Residual variance	
Residuals	37	.26609076	.00697921	
Variables in the model				
	B	SEB	T-RATIO	Approx. prob.
ARI	.9875134	.02084598	47.371882	.000000
EMP	-.00000400	.0001116	- 3.583352	.00097290
Constant	4.464375	.49147294	9.083664	.0000

(Analysis will be applied to the natural logarithm of the data).

$R^2 = .97$

Table (13): Final Parameters Estimates, Model (8) Life Expectancy at Birth (Multivariate analysis)

Number of residuals	23			
Standard error	.03391799			
Log likelihood	46.271318			
Analysis of variance				
DF	adj. Sum of squares		Residual variance	
Residuals	20	.02397662	.0115043	
Variables in the model				
	B	SEB	T-RATIO	Approx. prob.
ARI	-.4855326	.27235153	-1.78274	.08981523
SAL	.0000023	.00000035	6.75729	.00000143
Constant	4.0296242	.01445524	278.76559	.00000000

Analysis will be applied to the natural logarithm of the data.

$R^2 = .67$

Table (14) Predicted Values for Economic Indicators * from 2001 to 2020**

Year	Employment**	Domestic product*	Investment*	Salary*
2001	17608	51840	73837	96578.7
2002	17784	52358	74575	97544.0
2003	17962	52882	75321	98519.0
2004	18142	53411	76074	99504.0
2005	18323	53945	76834	100499.0
2006	18506	54484	77602	101504.0
2007	18691	55029	78378	102519.0
2008	18878	55579	79162	103544.0
2009	19067	56135	79954	104579.0
2010	19258	56696	80754	105625.0
2011	19451	57263	81562	106681.0
2012	19646	57836	82378	107748.0
2013	19842	58414	83202	108825.0
2014	20040	58998	84034	109913.0
2015	20240	59588	84874	111012.0
2016	20442	60184	85723	112122.0
2017	20646	60786	86580	113243.0
2018	20852	61394	87446	114375.0
2019	21061	62008	88320	115519.0
2020	21272	62628	89203	116674.0

*** Economic indicators are calculated from year 2001 to 2020 assuming that these indicators will be increased by 1% yearly.

** Thousand Employees

* Million pounds

Table (15): Predicted Values for CBR and CDR Using Multivariate analysis

Year	CBR LCL	CBR Fit	CBR UCL	CDR LCL	CDR Fit	CDR UCL
2000	24.01549	26.06795	28.2947	5.46710	6.51100	7.75423
2001	25.17555	27.21617	29.42219	5.39508	6.41974	7.63901
2002	24.59481	26.58955	28.7461	5.38997	6.41396	7.63250
2003	24.25689	26.22543	28.3537	5.46070	6.49846	7.73343
2004	24.0813	26.03691	28.1512	5.53110	6.58258	7.83394
2005	23.32870	25.96876	28.9075	5.25176	6.65095	8.42292
2006	22.79293	25.89607	29.4217	5.07761	6.70606	8.85677
2007	22.36929	25.81933	29.8015	4.95270	6.74906	9.19695
2008	22.01615	25.73895	30.0913	4.85571	6.78104	9.46979
2009	21.7174	25.65531	30.3152	4.77602	6.80306	9.69040
2010	21.44291	25.56864	30.488	4.70760	6.81606	9.86887
2011	21.2019	25.47934	30.6207	4.64672	6.82095	10.01251
2012	20.98054	25.38755	30.7203	4.59098	6.81854	10.12692
2013	20.77675	25.29362	30.7925	4.53899	6.80988	10.21692
2014	20.58647	25.19765	30.8417	4.48941	6.79537	10.28577
2015	20.40724	25.09982	30.8714	4.44148	6.77563	10.33647
2016	20.23710	25.00028	30.8846	4.39464	6.75122	10.37151
2017	20.07451	24.8919	30.8834	4.34851	6.72266	10.39302
2018	19.91821	24.79667	30.8699	4.30284	6.69041	10.40279
2019	19.76708	24.69274	30.8458	4.25720	6.65459	10.40203
2020	19.62038	24.58760	30.8123	4.21169	6.61586	10.39241

10- Model Checking:

The most important step in model building is to check the adequacy of the model and to asses its goodness of fit. First we calculated R^2 for each model. The value of R^2 is very high for all univariate and multivariate models. The only moderate value for R^2 is shown for the model of the life expectancy at birth which is .676. Second we get the plot of the sample autocorrelation function (ACF) for the error and their probability limits. Figures (9) to (12) show these plots. The residual ACF is acceptable since Box-L Jung statistic is not statistically significant at any lag.

11- Results and Conclusion:-

Table (15) shows the predicted values for crude birth rate and crude death rate using multivariate analysis. Table (8) shows the predicted values for crude birth rate and crude death rate using univariate analysis. Cairo Demographic Center (Makhloof, Hesham 2000) got the projections for crude birth rate and crude death rate according to three assumptions; low average and high for the period (1996-2021). The predicted values of the demographic center are more

Table (16): Predicted Values for IMR and LEB Using Multivariate Analysis

	IMR				LEB		
Year	LCL	Fit	UCL		LCL	Fit	UCL
2000	21.586	25.5937	30.3449		65.01596	70.3888	76.2057
2001	21.0938	24.9973	29.6231		62.6733	67.8073	73.3619
2002	19.6710	24.9905	31.7485		64.2597	70.6400	77.6538
2003	18.6500	24.9796	33.4573		64.3830	70.8012	77.8592
2004	17.8339	24.9646	34.9463		64.5076	70.9644	78.0674
2005	17.1475	24.9465	36.2928		64.6334	71.1296	78.2787
2006	16.5512	24.9245	37.5339		64.7604	71.2968	78.4930
2007	16.0221	24.8985	38.6925		64.8887	71.4662	78.7103
2008	15.5453	24.8685	39.7833		65.0183	71.6376	78.9307
2009	15.1106	24.8347	40.8164		65.14918	71.81108	79.1542
2010	14.7106	24.7970	41.7991		65.2814	71.9868	79.3810
2011	14.33965	24.75544	42.7368		65.4149	72.1647	79.61101
2012	13.9934	24.7102	43.7369		65.5498	72.3449	79.8444
2013	13.6693	24.6621	44.4954		65.6861	72.5272	80.0809
2014	13.36371	24.6104	45.3222		65.82360	72.7119	80.3210
2015	13.07453	24.55506	46.1165		65.96259	72.8989	80.5646
2016	12.79988	24.4961	46.8801		66.10299	73.0883	80.8116
2017	12.5381	24.4336	47.6147		66.24481	73.2741	81.0623
2018	12.2882	24.3677	48.3214		66.3880	73.4741	81.3165
2019	12.0482	24.2973	48.9996		66.5328	73.6708	81.5746
2020	11.8178	24.2234	49.6520		66.6702	73.8695	81.8364

Consistent with the multivariate analysis results than with the univariate analysis results. Table (17) and Table (18) illustrate the predicted values for crude birth rate and crude death rate issued by Cairo Demographic Center (Makhloof, Hesham 2000).

Table (17)
Crude Birth Rate Estimates for the period
(1996-2021) According to the Three Assumptions

Years	Low	Average	High
1996-2001	26.2	27.6	28.3
2001-2006	24.1	25.6	26.8
2006-2011	22.2	23.9	25.1
2011-2016	20.3	21.8	23.0
2016-2021	18.4	19.9	21.1

Table (18): Crude Death Rate Estimates for the Period
(1996-2021) According to the Three Assumptions

Years	Low	Average	High
1996-2001	7.1	7.2	7.2
2001-2006	6.5	6.5	6.6
2006-2011	6.1	6.1	6.1
2011-2016	6.0	5.9	5.9
2016-2021	6.0	5.8	5.8

The predicted values of crude birth rates of the Cairo Demographic Center are more optimistic than the multivariate analysis results, see table (15) with table (17), we noted that there is steadily decrease in the crude birth rate in Cairo Demographic Center predicted values as well as the multivariate predicted values. But the lower bound of the multivariate results for the crude birth rate for the time period 2018-2020 is 19.7 per thousand which is very near to the demographic center predicted values for the time period 2016-2021 according to the average assumption (19.9%)

Also, the predicted values of crude death rates of Cairo Demographic Center are optimistic than the multivariate analysis results. Comparing table (15) with table (17), we noted that there is very slow decline in the crude death rate in the demographic center predicted values as well as the multivariate predicted values. But the lower bound of the demographic center results are coincides with the predicted values of the multivariate analysis. As for example the predicted values for crude death rates for the time period 2018-2020 is 6.6 per thousand which is very near to the demographic center predicted values for the time period 2016-2021 according to the lower assumption (6.0%).

Table (16) shows the predicted values for infant mortality rates and life expectancy at birth using multivariate analysis. Table (9) shows the predicted values for infant mortality rates and life expectancy at birth using the univariate analysis. The predicted values of the demographic center are more consistent with the multivariate analysis results than with the univariate analysis results. Table (19) illustrates the predicted values for life expectancy at birth issued by Cairo Demographic Center (2000) for males and females separately.

Table (19): Life Expectancy at Birth by Sex (1996-2021)

Years	Males	Females
1996	63.78	65.37
2001	65.63	67.45
2006	67.25	69.22
2011	. 68.66	70.74
2016	69.89	72.06
2021	71.12	73.20

The predicted values for life expectancy at birth for males and females are 71.2 and 73.20 years respectively for the year 2021. The predicted value using the multivariate analysis is 73.8 years which are very near to the value for females. Table (16) illustrates the predicted values for infant mortality rates for the time period (2000-2020). The predicted values for infant mortality rate show steady but slow decline which coincides with the original data.

المراجع العربية :

1- والتى فاندل ، "السلاسل الزمنية من الوجهة التطبيقية ونماذج بوكسى – جينكيز " تعريب الدكتور/ عبد المرسى حامد عزام ، دار المريخ ، الرياض ، السعودية ، 1992.

2- مخلوف،هشام وآخرين ، إسقاطات السكان المستقبلية لمحافظات مصر لأغراض التخطيط والتنمية 2001 – 2021 ، المركز الديموجرافى بالقاهرة ، الجزء الاول : إجمالى الجمهورية . سبتمبر 2000.

References:-

1- Abraham Bovass & Ledolter Johannes (1983) Statistical Methods for Forecasting. John wily & sons. Inc.

2-Hussein M. A (1993). "Time series Analysis and forecasting Both Fertility and Mortality levels in Egyptian until year 2010". The Egyptian Population and Family Planning Review, Vol 29, No.2.

3-Hussein M.A and Mahgoub Y.M. (2000) "Time Series Analysis For Forecasting Mortality in Egypt" The Egyptian Population and Family Planning Review, Val 33, No.1

4-Mahgoup Y.M (2002). "Recent Trends in Fertility Analysis "The Egyptian Population and Family Planning Review, Vol 35, No.1