Fitting a Theoretical Modle to The Waiting Time to First Conception, with an Application to sudan

Mona Khalifa

Introduction

An important concept in the study of fertility is to divide the interval between each two successive live births into subintervals. The distinction between these subdivisions is based on the fact that different factors affect the length of each of them. The length af a birth interval is thus seen as being the sum of three subintervals:

(i) A period of post partum amenorhoea immediately following the birth initiating the interval. During this time, the mother is in a state of nonsusceptability to conception. The length of this period is affected mainly by the duration and intensity of breastfeeding.

(ii) A period of waiting time to conception. During this period the married woman may be using a contraceptive or may be exposed to the 'risk' of conception. The length of the period of exposure is determined by the 'fecundability' of the couple. Fecundability being defined as the probability of conception per month of exposure, in the absence of any deliberate action aiming at the delay or prevention of a next birth i.e. contraception.

(iii) A period of gestation. Although the duration of pregnancy varies among women and - for the same woman - by birth, it is realistic to assume a constant period of nine months:

To these three subintervals we add the time lost by one or more abortions (pregnancy wastages) that may by observed during the interval. It is clear that in the cese of the first birth interval i.e. the interval between marriage and the delivery of the first birth, the post partum sub-interval does not exist.

During her life, a woman may start a waiting time episode immediately after marriage, or after completing a period of post partum amenorrhea associated with a birth (or an abortion) or after stopping the use of a contraceptive. She is considered to be 'exposed' as long as she leads a normal married life with no use of contraception. She leaves this state by bcoming pregnant. The probability of conception for an exposed woman during a specific month (i.e. fecundability) is obviously a chance variable. But, it is also a function of biological and behavioural characteristics of the couple. As the principal determinant of the length of waiting time, fecundability becomes an important foctor in determining the length of the whole birth interval and hence a determinant of the fertility level of a population. This is especially the case in societies with little or no use of family planning.

However, in studies dealing with the estimation of fecundability a distinction must be made between three terms. 'Total fecundability' includes all conceptions, while 'recognizable fecunbility' deals with conceptions after excluding those that ended before the pregnancy was recognised i.e. within a few weeks. Effective fecundability' is the term used when dealing with conceptions that will end live birth. In this case the proportion of conceptions that were lost (aborted) after the pregnancy was recognised is excluded.

Since direct observation of the distribution of total fecundability in a population is not possible, we often estimate recognisable or effective fecundability. This is estimated indirectly through information about the wating times to conceptions or to birth.

Among other methods, demographers used models to fit theoretical functions to waiting time distributions. This was done for the purpose of smoothing the data that showed irregularities and to reduce the data to a limited number of parameters which describe the process adequately. When models are fittef to the birth interavl, additional data are required. These include the distribution of the components of the birth interval especially the duration of the post partum nonsuesc ptable period and pregnancy wastage. Several assumptions have to be made in order to determine the exact starting point of exposure. Nevertheless, when the distribution of waiting time to conception is used, no detailed data are required and no assumptions are necessary. This is because the only information required is the distribution of the period of exposure.However, except at the first birth interval (i.e. formarriage to first birth), estimation is very problematic because it is difficult to determine the exact end of the period of non-susceptability which marks the beginning of exposure. This situation attracted demographers to use the distribution of the first conception interval rather than that of the higher order births for the purpose of estimaying measures of fecundability. A major disadvantage, however, is that the estimates relate to the beginning of the married life only. However, recent studies have been able to show that the estimates based on the first conception interval are not very much different than estimates based on higher order conceptions once the effect of breastfeeding is controlled (Goldman et al 1987).

In the peresent paper we discuss the development of methods used for fitting models to the distribution of intervals from marriage to first conception and the method of deriving estimates of fecundability. We examine the possibility of using such procedures in connection with different types of demographic data. We illustrate the procedure by using data from the sudan Frility Survey, 1979.

The Model

First we assume that we have data from a prospective study of newly married, non contracepting women, none of whom were lost to the observation which continued for a long time. For each women, the number of months of exposure is calculated including the month when first conception occurs. We also assume that the fecundability of each wife remains constant during the period of observation.

Two measures of fecundability may be estimated. First, one can derive an estimate of the monthly probability of conception which is indicative to the population sbiological copability. this is done be observing the proportion of newly married women who conceive during the first month of exposure. The second approach is to observe, for the newly married women,the interval to conception which is a reflection of their fecundability. As the parameters of the distribution underlying the set of observations are estimated, it becomes possible to extend the use of the model to other situations e.g censored data. These are retrospcective data sets in which some of the women have not yet conceived when the information on the length of waiting time were collected. It also becomes possible to analyse data sets suffering from reporting errors.

First we assume a non realistic - but simplifying - situation of homogeneity. We assume that all of the observed women have an identical and constant monthly probability of conception i.e. fecundability = p. The probability that the waiting time is (x) months for a woman is equal to the probability that conception in the month number (x) after marriage. This situation is equivalent to a sequence of Bernoulli trials where the probabilities of conception in the months of exposure are given by a negative binomial distribution. Here, we consider the number of failures encountered before the first success = x. The distribution defined by these probabilities is said to be Geometric since the probabilities are terms in a geometric sreies.

The probability of conception (i.e. success) during the first month after marriage is p, during the second month is p (1-p), during the third month is p(1-p)2 and so on. This means that the unconditional probability of conception in the first month is at its maximum (P) and falls gradually as x increases. The parameter of the geometric seried is q (the probability of failure).

$$f_x(x) = p\,q^{x-1} \qquad x = 1, 2, \ldots$$
$$= f(x-1)\,q \qquad x > 2 \qquad (1)$$

In this case the expected number of months of waiting time is the mean of the geometric distribution i.e. $E(x) = 1/p$ and the variance is ver $(x) = q/p^2$ (Lindgren, 1962). This means that if the fecundability of all the womwn is $p = 0.2$, the mean waiting time for conception is five months after marriage.

On the other hand, the probability of not yet having conceived is the probability of 'failure' in each of the preceding trials.

$$pr\ x \underset{>}{\ } x-1 = Q_x(x-1) = q^{x-1} \qquad x>2 \qquad (2)$$

It becomes possible to define a conditional accurance rate wich is the probability of the conception occurring at time x, given. that it has not occurred before.

$$o_x(x) = \frac{f_x^{(x)}}{Q_x(x-1)} = \frac{pq^{x-1}}{q^{x-1}} = p \qquad (3)$$

This means that the expected probability of conceiving in a month given that conception did not occur before is equal to the probability of conception in the first month = p, a result which is consistent with the assumptions. In every month, the propoprtion of women expected to conceive is p of those who had not conceived before i.e. at risk.

The above statistics may be derived using the moment generating function of the geometric distribution which is

$$C(s) = \sum_{x=o} p\ e^s\ (q\ e^s)^{x-1} = \frac{pe}{1-q^{es}} \qquad (4)$$

and the k (th) moments may be calculated by differentiating the function k times with respect to s and evaluating the derivative at s=o.

For a specific woman, fecundability depends on many factors of a biological or behavioural nature. Therefore it may be assumed that the value of p varies randomly during the period of observation. The value of p at a given month is a value selected from a distribution and is independent of the woman's previous p values. This is similar to sampling with replacement. Thus, we may view fecundability as being constant throughout the period of observation and equal to the mean of the distribution $E(p) = \bar{p}$. The above results will therefore remain applicable.

On the other hand, empirical research has confirmed the existence of differences in the fecundability between women in the population. Fecundability varies neacessarily with age as well as other factors. Possible explanations have been offered including genetic, physiological, cultural factors in addition to socio-economic characteristics of the couple(Jain 1969a, 1969b; kallan and Udry 1986). Therefore, the assumption of the homogeneous population must be relaxed.

Under the - more realistic - assumption of heterogeneity, the population under observation is regarded as being made up of several subgroups of newly married women each of which is homogeneous with respect to fecundability. For each woman, p is constant over time or varies randomly. But, p for different women is not identical and varies systematically. we assume that p Has a probofility density function f(p) from which a specific value e.g p_i refers to the fecundability of the i th woman. This situation is equivalent to a multiple sequence of Bernoulli trials. Each sequence has a constant probability of success p and is specific to one woman. The combined sequences becone a model decribing the heterogeneous population with varying probabilities f(p).

The form of the distribution of (p) in human Populations is not exactly known. But, researchers realised that in order to formulate - theoretically - a set of waiting times to conception, it is necessary to assume an underlying distribution for f (p). Based on the idea that fecundability can only range between zero and one. Henry suggested the use of a Beta distribution (Pearson Type I curve) Henry, 1961, 1964). The Beta curve takes different shapes depending on the values of its parameters a and b. This suggestion was first used by potter and parker to develop a waiting time model which was used to predict the time required to conceive for a group of American brides (potter and parker, 1964). Soon, the specific distribution assumption was generalised by Sheps, and measures of fecumdability were formulated that can apply to a frequency distribution of fecundability of any shape (Sheps 1964, Sheps and Menken 1973). In the following paragraphs we present the model in its general form and when a Beta distribution is assumed. The model is used to derive the mean monthly probability of conception for the population $\bar{P}$ and the moments of the waiting time distribution i.e the mean waiting time $\bar{w}$, its variance and other moments.

In general, the expected proportion of women conceiving during the first and second months of x eposure are

$$p(1) = \int_0^1 p\ f(p) = \bar{p} \qquad (5)$$

$$p(2) = \int_0^1 q\ p\ f(p)\ dp = \int_0^1 p(1-p)\ f\ (p)\ dp$$

$$= E(p) - E(p^2)$$

$$= E(p) - (Ep)^2 - \sigma_p^2$$

$$= \bar{p} - (\bar{p}^2 + \sigma_p^2 \qquad (6)$$

This shows that in the heterogeneous population the expected proportion of women to conceive during the first month of exposure is equal to the mean of the fecundability distribution. However, the proportion of the total population, expected to conceive in the second month is smaller. It is also smaller than that expected in the homogeneous population which was given as $\bar{p}\ \bar{q}$ since.

$$\bar{p}\ (1-\bar{p})\quad \sigma_p^2 \qquad < \quad \bar{p}(1-\bar{p})$$

This result is obvious because women who conceive immediately are those with high values of p. As time goes on, women of high fecundability become pregnant and are therefore not subject to the risk. The remaining women are those with lower fecundability and thus smaller proportions of them conceive. This proportion decreases as x increases. The probability of conception in month x is given by

$$p(x) = \int_0^1 p(1-p)^{x-1}\ f(p)\ dp \qquad (7)$$

The conditional probability of conceiving in month x equals the mean fecundability of the women who are still exposed at month x. As shown before this mean decreases as time goes on.

$$O_x(x) = \int_0^1 p\, q^{x-1} f(p)\, dp / O_x(x-1)$$

$$= E(p|x) \qquad (8)$$

It follows that the rate of occurrance i.e the proportion of women conceiving among those still exposed decreases as x increases until it reaches the minimum fecundability of the population .

When a Beta distribution is assumed to describe the fecundabilty variation in the population the corresponding formulae are as follows since

$$f(p) = \frac{1}{B(a,b)}\, p^{a-1} q^{b-1} \qquad a,b > o$$

where,

$$B(a,b) = \int_o^1 p^{a-1} q^{b-1}\, dp$$

$$= \frac{\Gamma a\, \Gamma b}{\Gamma a + b} \qquad (9)$$

from (5)

$$p(1) = \mu_p = \frac{a}{a + b} \qquad (10)$$

from (7)

$$p(x) = \int_0^1 p\, q^{x=1}\, \frac{1}{B(a,b)}\, p^{a-1} q^{b-1}\, dp$$

$$= \frac{1}{B(a,b)} \int_o^1 p^{a} q^{b+x-2}\, dp$$

$$= \frac{B\,[\,(a+1)\,,\,(b+x-1)\,]}{B(a,b)} \qquad (11)$$

from (8)

$$O_x(x) = \frac{\int_o^1 p\, q^{x-1} f(p)\, dp}{\int_o^1 q^{x-1} f(p)\, dp}$$

$$= \frac{\int_o^1 p^{a}\, q^{b+x-2}\, dp}{\int_o^1 p^{a-1}\, p^{b+x-2}\, dp}$$

$$= \frac{B\,[\,(a+1)\,,\ (b+x-1)\,]}{B\,[\,a\,,\ (b+x-1)\,]} = \frac{a}{a+b+x-1} \qquad (12)$$

Themean, mode and variance of the fecundability distribution are those known for the Beta distribution to be

$$\text{mean} \qquad \bar{p} = \frac{a}{a+b}$$

$$\text{mode} \qquad p' = \frac{a-1}{a+b-2} \qquad (13)$$

$$\text{variance} \qquad p^{2} = \frac{ab}{(a+b)^{2}\,(a+b+1)} \qquad (14)$$

To find the moments of the waiting time . distribution we must include fecund women only i.e those who have had a first birth. This means that sterile women with p=o are excluded. We also must assume the the observation extends for a long time so that all women are given a chance to coneive. Our population is assumed to be finite with n possible values of p. As mentioned before, the time at which conception accurs for a woman is dependent on the value of her constant fecundability pi . Also, we note that the distribution of x, (the month of conception on the waiting time) is determined by the expected waiting time for each woman, given her level of fecundability. therefore we define a new variable (waiting time to conception) $w_i = \frac{1}{p_i}$.

The mean and variance of the waiting time, for each woman is known from (1) to be

$$E(x|p_i) = \frac{1}{p_i} = w_i \quad \text{and} \tag{15}$$

$$\text{var}(x|p_i) = q_i|p_i^2 = (1-p_i)|p_i^2 = W_i^2 - W_i \tag{16}$$

To determine the mean of the waiting time distribution for the whole population we have:

$$E(x) = \frac{1}{n}\sum_{i=1}^{n} E(x|p_i) = \frac{1}{n}\sum_{i=1}^{n} W_i = \bar{W} \tag{17}$$

and we note that

$$\bar{W} = E\left[\frac{1}{p_i}\right] = 1/H_p$$

where Hp is the harmonic mean of p.

To determine the variance of the distribution of waiting time

$$\text{var}(x) = Ex^2 - (Ex)^2 = E[E(x|p_i)]^2 - \{E[E(x|p_i)]\}^2$$

$$= E[\text{var}(x|p_i)] + E[E(x|p_i)]^2 - \{E[E(x|p_i)]\}^2$$

$$= E[\text{var}(x|p_i)] + \text{var}[E(x|p_i)]$$

$$= E\left[w_i^2 - w_i\right] + \text{var}\, w_i$$

$$= EW_i^2 - EW_i + \sigma_w^2$$

$$= \sigma_w^2 + (\bar{W})^2 + \sigma_w^2$$

$$\text{var}(x) = \bar{W}(\bar{W} - 1) + 2\sigma_w^2 \tag{18}$$

The above results show that :

(i) The expected waiting time to conception in the heterogeneous population is equal to the reciprocal of the harmonic mean of the fecundabilities. Because the harmonic mean is less than the arithmetic mean for a distribution, $\bar{w}$ (the expected waiting time to conception) is longer than $1/\bar{p}$ which is the mean wating time expected from the arithmetic mean of the distribution.

(ii) The variance of the waiting time in a heterogeneous population is greater than that in a homogeneous population having the same mean delay which is $q/\bar{p}^2 = 1-\bar{p}/\bar{p}^2 = \bar{w}^2 - \bar{w} = \bar{w}(\bar{w} - 1)$. The amount of difference is $2\sigma^2_w$ as seen form (17).

When p is assumed to follow a Bata distribution, the moments of the waiting time distribution can be obtained as follows. Define $w = 1/p$ which will have a probability distribution function known as a pareto distribution

$$g(w) = \frac{1}{B(a,b)} \quad \frac{(w - 1)^{b-1}}{(w)^{a+b}} \qquad 1<w<\infty \quad (19)$$

and its moments are

$$E(w) = \frac{(a+b-1)\ (a+b-2)\ \ldots\ (a+b-w)}{(a-1)\ (a-2)\ \ldots (a-w)}$$

then,

$$E(w) = \frac{a+b-1}{a-1} \qquad (20)$$

$$\sigma_w^2 = \frac{b(a+b-1)}{(a-1)^2 \quad (a-2)} \qquad (21)$$

Substituting in the general model we find that

$$E(x) = \bar{w} = E(w) = \frac{a + b - 1}{a - 1} \qquad (22)$$

$$var(x) = \bar{w} \ (\bar{w} - 1) + 2 \ \sigma_w^2$$

$$= \frac{(a + b - 1)}{(a-1)^2} - \frac{a + b - 1}{a - b} + \frac{2b(a + b - 1)}{(a-1)^2 \quad (a-2)}$$

$$= \frac{ab(a + b - 1)}{(a-1)^2 \quad (a-2)}$$

$$= a \quad \sigma_w^2 \qquad (23)$$

note that the mean is not defined unless $a > 1$ and the variance is not defined unless $a > 2$

sheps and Menken demonstrate numerically that it is not enough to define the distribution of fecumdabi lity in terms of its arithmetic mean only. This is because - assuming a Beta distribution we find that

$E(p) = a / (a+b)$ .....(10) and $H_p = (a-1)/(a+b-1)$... (24)

so that,

$$H = \frac{(a-1) \ E(p)}{a-E(p)}$$

and thus for a specific $E(p)$, the value of the hermonic mean (and consequently $\bar{w}$) varies according to the shape of the Beta curve determined by the value of a. Therefore, in the heterogeneous case, it is more informative to estimate E(x) i.e $\bar{w}$ and H_p or the whole distribution.

Also, it is important to distinguish between these two types of measures i.e. the mean probability of conception p which is the proportion of women who conceive within the first month of exposure and the mean waiting time to conception $\bar{w}$ or H_p. This is because the average monthly probability of conception (p) cannot be derived from measures based on the average waiting time E(x) without the specification of the underlying function of the heterogeneity distribution. For the same reason, the average waiting time to conception cannot be calculatec from the monthly probability of conception. This is to say that one type of measure cannot be calculated from the other without additional knowledge about the true distribution of fecundability.

Fitting procedures

(1) Method of moments

The theoretical expressions for the mean and variance of the waiting time distribution are equated to the observed corresponding values. We need these two statistics for the estimation of the two unknown parameters of the Beta distribution (a,b). Let m and s^2 be the empirical statistics, then from (21) and (22) given that $E(x) = \frac{a+b-1}{a-1}$

$$\text{and } var(x) = \frac{ab\ (a+b-1)}{(a+1)^2\ \ a-2)}$$

we find by equating to m and s^2 that

$$\hat{b} = (m-1)\ (\hat{a}-1) \qquad (25)$$

and

$$\hat{a} = \frac{2s^2}{s^2 - m^2 + m} \qquad (26)$$

But, these estimatimators are shown to be biased (Sheps and Menken, 1973). The bias arises because they are ratios of two correlated quantities. However, these estimators are asymptotically consistent i.e. they approach the parameter valuse as n becomes large. A critical analysis of this method by Majumdar and Sheps (Majumdar and Sheps, 1970) showed that the moment estimates of a and b are reliable only in a specified range of a. The estimates of a <2 are not acceptable, and that the asymptotic relative efficiency of the moment estimates of b is poor for a <4.5. Nevertheless the moment estimates are easy to calculate and therefore are preferred long as the parameters lie within the specified range and the sample size is large. Otherwise maximum likelihood estimators are preferred.

(2) Maximum likelihood estimators

They result from equating the observed proportions conceiving in each of the first two months to the expected values of these proportions (Sheps , 1964). Assume that in a sample of n women, n_1 conceived in the first month and n the seond month. The mean fecundability is $\bar{p}$ and variance σ_p^2.

From (5), the probability of conceiving in the first month = $\bar{p}$

From (6), the probability of conceiving in the second month= $\bar{p}\,\bar{q} - \sigma_p^2$

The probability of not conceiving in the first or second $= 1 - (\bar{p} + \bar{p}\,\bar{q} - \sigma_p^2)$

$$= \bar{q} + \sigma_p^2$$

The likelihood function is

$$LF = \bar{p}^{\,n_1} \cdot (\bar{p}\,\bar{q} - \sigma_p^2)^{n_2} \cdot (\bar{q} + \sigma_p^2)^{n-n_1-n_2}$$

By setting the partial derivatives of the logarithm of the likelihood function equal to zero and solving for p and ^{2} we find that :-

(i) $\hat{p} = n_1/n$ which is the observed proportion of women who conceive in the first month c,

(ii)
$$\hat{\sigma}_p^2 = \frac{n_1(n-n_1)}{n^2} - \frac{n_2}{n}$$

$$= \frac{n_1}{n} \times \frac{n-n_1}{n} - \frac{n_2}{n}$$

$$= \hat{C}_1 (1 - \hat{c}_1) - \hat{c}_2$$

In this case an estimate of the mean fecundability is taken to be equal to the observed proportion of women who conceived in the first month and the variance is calculated using the same proposition together with the proportion who conceived during the second month . No assumption is therefore necessary regarding the shape of the distribution . However, the accuracy of the estimates depend on the quality of the collected data.

From the above discussion it seems that in order to use the model for analysing waiting time distributions, we have to assume that the fecundability of each women remains constant = p, that conception is a random event and to use a latge sample of couples . A Beta distribution can be used to describe the frequency of the different fecundability values in the population . Although, the model is viewed as an approximation of the true distribution of conception delays, it can be used to - roughly - describe the process, especially in cases of low quality data .

An application of the model to Sudanese data on first birth interval

To illustrate the application of the above procedure, we use data from the Sudan Fertility Survey of 1979 (SUDFS) whic is part of the World Fertility Survey programme.

The available information on the first birth interval consists of the distribution of the time between marriage and first live birth . In order to calculate from this information, a distribution of waiting time to conception one needs to subtract two more additional functions. These are (i) the pregnancy duration distribution and (ii)the incidence and timing of late spontaneous abortions.

Using parameters (on pregnancy duration and foetal mortality) derived from historic populations, Bongaarts was able to calculate the distribution from marriage to first birth which took into account the two additional functions (Bongaarts, 1975). He calculated the mean and variance of the distribution of first births for five populations and found that his model provides a good fit to the observed data . Another important finding was that the cofficient of variation in fecundability is almost constant, close to 0.56 in spite of the large differences in the means. Bongaarts then developed a simple method for the estimation of the mean fecundability of a population from the distribution of the interval between marriage and first birth . From this distribution a simple statistic is calculated which is used to locate the mean fecundability from a reference table. The needed statistic (S) is the proportion - of all births - that occurs during the first year of marriage (excluding birth in the months 0 Through 8). The Sudanese data may be handled in this way to arrive at estimate of the mean and variance of fecundability. We may then use a Beta distribution to describe the fecundability pattern of the population.

However, in order to use survey data some methodological issues have to be considered (Goldman et al 1985). First, we note that the dates of events were (as in most demographic surveys) coded in monthly intervals. Therefore, if the birth tokes place tow wards the end of the ninth month after marriage, the interval will be calculated as nine months even if the marriage date was at the beginning of the first month. This is because an event occurring at any time during a specific month is coded as accuring in the middle of that month. This difference is acceptable for most purposes but it produces biased results when dealing with short time durations e.g months (Goldman et al 1984) . To overcome this difficulty we have used the smoothed distribution by calculating three months moving averages.

As we have seen before, the model is model is applicable to prospective data in which each woman is given an adequate time to conceive, during a long observation period. On the othamd hand retrospective surveys - of the WFS typr - deal only with information collected from women at a specific point in time. At the time of the interview some women would not have conceived yet. If given more time, some of them would eventually conceive . Consider a woman who is interviewed one year after her marriage and was reported to have not given birth and was therefore classified as having an open interval. However this woman may have her first birth after the interview and the first interval will be closed. This type of information collected by retrospective surveys is said to be 'censored'. If 'censored' data are analysed directly, the resulting estimates will be biased.

To minimise this type of bias, we exclude from our analysis women who were married for less than three years. Thus, all the women included would have had a period of three or more years in order to have a first delivery. However, we cannot include all the marriages that took place three or more years before the survey. this is because the further we go back in time, the data are subject to more error resulting from memory laps. We therefore limit our analysis to marriages that occurred 3-8 years before the surv swrvy i.e 1971-1976. We thus avoid the censoring bias and capture the most recent period.

Also, a major problem in the study of fecundability is that the women included in the analyiss must be non users of contraception. Otherwise the analysis will not reflect the natural fecundability. In the Sudan, the use of contraception is generally very low. It is extremely unikely that contraception is used by women before their first birth. If that was not the case our analysis would have been restricted to non-contracepting brides.

We use for the analysis a distribution of women according to the month of girst birth fiven thaat the conception took place during the first marriage. Because these women had had a first birth, they are expeced to be more fecund that a cross-section of women married during the same period, which will - naturally - contain sterile women.

Results

The number of women available for this analysis is N = 469. The smoothed distribution of intervals from marriage to first birth shows that the proportion of births that occurred in the first year of marriage (excluding firths in months 0 through 8) is equal to 22.9/ (see table 1 in the appendix). This value corresponds to a mean fecundability value of $\bar{p}$ = 11 from Bongaarts reference table (Bongaarts, 1975). Since, the coefficient of variation for the fecundability distribution is almost constantly equal to 0.56, the variance of the Sudanese fecundability distribution is close to σ^2_p = .004 . We note that these values are at the very low limit of reported fecundabilities, in fact p = o.10 is the lowest value in Bongaarts' table.

Equating these two moment estimates to the mean and variance of a Beta distribution using equations (10) and (14) we can estimate the parameters. (a and b) by solving the simultaneous equations,

$$\bar{p} = \frac{a}{a + b} = .11$$

$$\sigma^{1\,2}_p = \frac{a + b}{(a+b)^2 \quad (a+b+1)} = .004$$

We find that $\hat{a}$ = 2.6 and $\hat{b}$ = 20.9.

To calculate the theoretical waiting time distribution, we not that the cumulative proportion of women pregnant by the end of month j, is

$$P(1+2+\ldots+j) = \int_0^1 f(p)\ \{1-(1-p)^j\}\ dp$$

$$= 1 - \int_0^1 \frac{1}{B(a,b)} \cdot p^{a-1} (1-p)^{b+j} dp$$

$$= 1 - \frac{B(a,b+j)}{B\ (a,b)}$$

where $B(a,b) = \Gamma a \ \Gamma b / \ \Gamma(a+b)$,

and $a = (a-1)\ !$

then,

$$\frac{B(a,b+j)}{B\ (a,b)} \quad \Gamma_{b+j} \cdot \Gamma_{a+b+j} \quad \Gamma_{b}$$

$$= \frac{(b+j-1)!\quad (a+b-1)!}{(a+b+k-1)!\quad (b-1)!}$$

when $j = 1$ this quantity is

$$= \frac{b!\ \ (a+b-1)!}{(a+b)!\quad (b-1)!} = b/a+b = c_1$$

when $j = 2$

$$= \frac{(b+1)!\quad (a+b-1)!}{(a+b+1)\quad (b-1)!} = (b+1)\ b/(a+b+1)\ \ (a+b)$$

$$= \frac{b+1}{a+b+1} \times c_1 = c_2$$

when $j = 3$

$$= \frac{(b+2)!\quad (a+b-1)!}{(a+b+2)!\quad (b-1)!}$$

$$= \frac{(b+2)\ \ (b+1)b}{(a+b+2)\ \ (a+b+1)\ \ (a+b)} = \frac{b+2}{a+b+2} \times c_2 = c_3$$

and so on.

We thus obtain the theoretical cumulative distribution of waiting time to conception (see table 2 in the appendix). By subtracting each consecutive values we get the density function.

$$p(j) = P(1+2+.....+j) - p(1+2+....+j-1).$$

We can now describe the distribution, that theoretically - fits the experience of waiting time to conception for Sudanese brides. The mean and variance of the distribution are calculated using equations (20) and (21) to be :

$\bar{W}$ = 14.1 months, σ^2_w = 313.5 this means that the waithing time to conception lies in the range 12.5 - 15.7 months (0.95 confidence level).However, it is seen from table 2 that the median of the distribution is at 606 months.

The above results show contrary to expectations - that the effective fecundability of Sudanese women calculated from the available data is low. Imagine a large population of couples who not use contracption immediately after marriage and all of whom conceive eventually andhave a first birth . According to the model, 11/ of the women conceive in the first month after marriage. Half of the population conceive before 6.6 months after marriage, and by the end of the first year, about 68/ do. By the end of the third, fourth and fifth years, about 92/, 95/. and 97/ of them conceive.

These estimates are lower than most published estimates besed on good techniques and reliable data. From British historical data a range of p = 0.18 and 0.31 was reported (Wilson, 1986) But, a lower range of fecundabilities was reported by Trussell (Trussell, 1977) to be 0.09 - 0.25, . The lowest fecundability mean was reported for the Bengalis to be 0.091 (Menken, 1975) which corresponds to 17 months of mean waiting time . The Sudanese data are known to be of low quality especially those concerning dates . However, the techniques by used WFS standard procedures have improved the data qulity than the level encountered in previous censuses and surveys (Rizgalla, 1985) In addition to the problems of data quality two possible explanations for the low estimated fecundability con be sidered . First, are the physiological complications associated with female circumcision (Rushwan, 1983) . These complications are known cauedelays in conception at the beginning of the married life. Second, is the low age at marriage. Fecundability is known to be a function of age (Jain, 1969 a). The published estimates of fecundability are those for women in their twenties (Bongaarts and potter, 1983)

In order to investigate the second possibility we selected, from our sample, the of women who married at the age of twenty or above, (N=110) . The same technique was applied to those women[1] and it was found that they have an average fecundability of 0.12. The waiting times to conception - theoretically - fit a Beta distribution with parameters a = 2.7 and b = 19.65 (see appendix tables 3.4). Table 3 shows a median waiting time of 6 months . The mean waiting time to conception is thus estimated as $\bar{w}$ =12.6 months with a variance

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(1) Jain, 1969 showed that it is valid to fit a Beta distribution to the subgroups of the population in the same way as was done for the entire population.

σ^2_w = 118.2 . This mens that the 95/ confidence interval is 11.9 - 13.3 months. It seemes that women who marry late have a better chance of conception and a shorter waiting time . This agrees with earlier findings by Jain c Jain, 1969a) who showed a similar pattern for a sample of Taiwanese women. Women who married before their sixteenth birthday took the longest time to conceive, the delay reduce with increasing age at marriage and remains almost constant for women who marry between ages 21 and 25 . We may - therefore - regard the estimates derived from the subsample of old marriers to be the highest for our population and to be comparable with the published estimate of mean fecundability. That is to say that a plausible estimate of mean fecundability for Sudanese women is p = 0.12 which corresponds to a mean waiting time of 12-13 months and a median of 6 months .

A comparative study was recently carried out to examine waiting times to conception in higher order birth intervals using WFS data including SUDFS (Goldman, Westaff and paul, 1987). The estimated median waiting time to conception for Sudan was calculated to be 11.5 months for women who have been married for 0.9 years. This estimate is the highest among the countries studies (Kenya, Lesotho, Syria) and about 26/ higher than their average whic was about nine months. Although this figure is not comparable with our previous estimates which related to the first birth interval only, it cofirms the low level of fecundability in this population in contrast with that found in other countries. Our analysis confirms that fecundability is low among Sudanese women at all stages of their reproductive life.

Conclusions

The above analysis has shown that by using the fitting procedure outlined,we were able to smooth the data on the first conception wait. The model was used on data with minimum censoring problems. As the distribution became defined, several statistics were made available that describe the process of first conception. It was shown that in spite of the high fertility rates of Sudan, the level of fecundability is very low. Further research will be needed to examine what appears to be a contradictory situation.

Appendix

Table (1)

The distribution of women by duration of first birth interval in months (N = 469)

month	proportion	cumulative proportion%	month	proportion	cumulative proportion
<8	2.4		25	2.3	68.0
9	8.6	8.6	26	1.5	69.5
10	7.0	15.6	27	1.1	70.5
11	7.3	22.9	28	1.4	71.9
12	6.4	29.3	29	1.5	73.4
13	5.5	34.8	30	.8	74.2
14	2.8	37.7	31	2.8	77.0
15	3.2	40.9	32	1.2	78.2
16	3.2	44.1	33	1.4	79.2
17	2.6	46.7	34	.9	80.5
18	2.7	49.4	35	2.0	82.6
19	2.8	52.3	36	1.5	84.1
20	2.2	54.7	37-	3.5	87.6
21	3.2	57.3	42-	3.4	91.0
22	1.8	59.4	48-	3.9	95.0
23	3.5	63.0	60+	2.6	97.6
24	2.7	65.7			
			Total	100	100

% excluding births occurring before 9 months.

Table 2

Theoretical waiting time to conception
(a = 2.6, b = 20.9) (all women).

month	cumulative proportion	proportion	month	cumulative proportion	proportion
1	11.1	11.1	31	90.1	.6
2	20.5	9.4	32	90.5	.4
3	28.6	8.1	33	91.0	.5
4	35.6	7.0	34	91.4	.4
5	41.7	6.1	35	91.8	.4
6	47.0	5.3	36	92.1	.3
7	51.7	4.7	37	92.5	.4
8	55.8	4.1	38	92.8	.3
9	59.5	3.7	39	93.1	.3
10	62.7	3.2	40	93.4	.3
11	65.6	2.9	41	93.7	.3
12	68.2	2.6	42	93.9	.2
13	70.5	2.3	43	94.2	.3
14	72.6	2.1	44	94.4	.2
15	74.5	1.9	45	94.6	.2
16	76.2	1.7	46	94.8	.2
17	77.8	1.6	47	95.0	.2
18	79.2	1.4	48	95.2	.2
19	80.5	1.3	49	95.4	.2
20	81.7	1.2	50	95.5	.1
21	82.8	1.1	51	95.7	.2
22	83.8	1.0	52	95.8	.1
23	83.7	0.9	53	96.0	.2
24	85.6	0.9	54	96.1	.1
25	86.4	0.8	55	96.3	.2
26	87.1	0.7	56	96.4	.1
27	87.8	0.7	57	96.5	.1
28	88.4	0.6	58	96.6	.1
29	89.0	0.6	59	96.7	.1
30	89.5	0.5	60	96.8	.1

Thable 3

The distribution of women by duration of first birth interval in tmonths, women married at age 20+ (N = 110).

month	proportion	cunulative proportion*	month	proportion	cumulative proportion
<8	1.7		25	2.3	72.3
9	5.8	5.8	26	-	72.3
10	6.9	12.8	27	2.9	75.2
11	11.6	24.3	28	-	75.2
12	8.7	33.0	29	1.2	76.3
13	7.5	40.5	30	1.2	77.5
14	4.0	44.5	31	4.6	82.1
15	5.2	49.7	32	1.2	83.3
16	4.6	54.4	33	1.2	84.4
17	-	54.4	34	-	84.4
18	2.3	56.7	35	1.2	85.6
19	3.5	60.1	36	2.3	87.9
20	1.2	61.3	37-	3.0	90.8
21	1.7	63.0	42-	3.5	94.3
22	2.3	65.4	48-	3.0	97.1
23	2.3	67.7	60+	1.2	98.3
24	2.3	70.0			
			Total		100

Table 4

Theoretical waiting time to conception (a = 2.7, b = 19.65) for women married at ages 20+

month	cumulative proportion	proportion	month	cumulative proportion	proportion
1	12.0	12.0	31	91.7	.5
2	22.0	10.0	32	92.1	.4
3	30.9	8.9	33	92.5	.4
4	38.2	7.3	34	29.9	.4
5	44.6	6.4	35	93.2	.3
6	50.0	5.4	36	93.5	.3
7	54.8	4.8	37	93.8	.3
8	59.0	4.2	38	94.1	.3
9	62.6	3.6	39	94.4	.3
10	65.8	3.2	40	94.6	.2
11	68.7	2.9	41	94.9	.3
12	71.2	2.5	42	95.1	.2
13	73.5	2.3	43	95.3	.2
14	75.5	2.0	44	95.5	.2
15	77.3	1.8	45	95.7	.2
16	79.0	1.7	46	95.8	.1
17	80.4	1.4	47	96.0	.2
18	81.8	1.4	48	96.2	.2
19	83.0	1.2	49	96.3	.1
20	84.1	1.1	50	96.4	.1
21	85.1	1.0	51	96.6	.2
22	86.1	1.0	52	96.7	.1
23	86.9	.8	53	96.8	.1
24	87.7	.8	54	96.9	.1
25	88.4	.7	55	97.0	.1
26	89.1	.7	56	97.1	.1
27	89.7	.6	57	97.2	.1
28	90.2	.5	58	97.3	.1
29	90.8	.6	59	97.4	.1
30	91.2	.4	60	97.5	.1

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