

**The Relational Gompertz Model in
Detecting the Recent Changes of
Fertility in Egypt
By: Mohamed Naguib Abdel Fattah**

Fertility is an important component of population growth in Egypt, especially as mortality has declined in a short span of time. Egyptian fertility trends during the last two decades have puzzled demographers and other (Bucht and Elbadry 1986). This surprising trends has occurred because of the steadily decline in fertility between 1966 and 1976 where total fertility rate (TFR) decreases from 6.33 to 5.6 births per woman, but this decline was followed by an increase, as the census of 1986 reveals a high TFR estimated to be 5.8 births per woman, while in the beginning of 1980's this rate was 5.27 births per woman.

A Sharp decline in fertility was observed in 1988, DHS estimated TFR to be 4.69 births per woman. The PAP/CHILD survey (1991) estimated TFR to be 4.6 births per woman, nearly the same as in DHS (1988), and DHS (1992) confirmed this decline in TFR to become 3.9 births per woman (Osheba, and Ahmed 1992). The contraceptives use who 47.8% DHS (1992), while it was about 37.8 DHS (1990).

This trend raise a question ? :
does this change real or not? or does really fertility was declining in the last two decades?

It was found from the literature that applying indirect techniques are very important tool for countries having detectives or uncompleted data. But these indirect techniques have certain assumptions when applying them, for example the P/F ratios which were developed by Brass (Brass 1968) was based on the main assumption that fertility has been constant in the past. So, this method seem inappropriate if there has been recent changes in fertility as in Egypt. Consequently, it seems necessary to seek same other techniques. One of this techniques is Gompertz relational model.

This model was designed for measuring the force of mortality in 1825 by Benjamin Gompertz, and was applied by Brass in 1974 to fertility data.

2- The Objective of the Research:

The main objective of this research is to detect the recent changes in fertility in Egypt using Gompertz's relational model.

3- Data Source:

The PAP/CHILD survey (1991), and DHS (1992) have provided information on the fertility behavior of the women and related aspects. The analysis of this chapter is based on the data collected in the maternity history section of the PAP/child (1991) and DHS 1992 individual questionnaire for ever married women aged 15-49 years. The number of women sampled was 9073 and 9978 respectively.

4- The Methodology:

Two methodologies will be considered in this research; first: an orthodox analysis of fertility; second: Gompertz relational model for detecting the recent changes in fertility.

For an orthodox analysis, a comparison of data for current fertility as total fertility rates, etc....., are used in the next part for the recent surveys starting from the eighties.

As regards the Gompertz model, it was found from the literature that applying indirect techniques are very important tool for countries having defective or incomplete data. For example P/F ratios which were developed by Brass (Brass, 1968), was based on the main assumption that fertility has been constant in the past. So, this method seems inappropriate in case there have been recent changes in marital fertility or those in age at

first marriage. In fact, this assumption is not appropriate for Egypt because the mean age at first marriage (SMAM) had increased

from 21.3 years in 1980 in to 22.36 years in 1991.

Consequently, it seems necessary to seek some other techniques to cope with the previous problem. One of these techniques is Gompertz relational model.

Gompertz's model was designed measuring the force of mortality in 1825 by Benjamin Gompertz, and was applied by Brass in 1974 to fertility data.

For fitting this model to fertility, the required data are the mean children ever born and age specific fertility rates. This technique relaxes the assumptions that fertility has been constant in the recent past or that the quality of reporting births does not vary with mother's age, which were required for the P/F ratios technique. In brief, the Gompertz relational model links two schedules of fertility through only two parameters which were developed by Zaba (1981). The equation for fitting the Gompertz relational model to current fertility is given by:

$$Z(X) - e(X) = \alpha + .48 (\beta - 1)^2 + g(X) \dots \dots \dots (1)$$

Where,

$$Z(X) = -\ln (F(X)/F(X+5))$$

$F(X)$ = is the cumulated age specific rate up to age x,
(representing the total fertility rate, up to age x)

α and β are constant corresponding to a particular set of fertility rates which measure the location and dispersion of the distribution respectively.

The function $e(x)$ and $g(x)$ are tabulated as standard values. For mean parities $P(i)$ and $Z(i)$ $\ln [-\ln (P(i)/P(i+1))]$, the above equation becomes:

$$Z(i) - e(i) = \alpha + .48 (\beta - 1)^2 + g(i) \dots \dots \dots (2)$$

Where

$P(i)$ is the mean parity for the age group i.

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This model is applied to births in the five year period preceding the survey. Strictly speaking, the resulting measures are the average rates as the women grow on age to move from the age groups 10-14, 15-19, 20-24 etc. The cumulated rates are then the mean parities for conventional age groups of the synthetic time period cohort.

To use this model, a computer program has been designed by the researcher under the name (Gomp1). It can be used (ISSR) and part of the output is enclosed in the appendix (1).

5- The Analysis

5.1 An Orthodox Analysis

The measures used in this section make use of two important of any individual woman's childbearing capability, viz the: Quantity or level, which expresses how many children the woman has had, and the tempo, which expresses how quickly she has had them. The central problem of fertility analysis is that these two dimensions can not be fully separated.

To analyses the fertility component, conventional (orthodox) demographic methods have been applied only at the National level. The results are presented in the first National report of the PAP CHILD survey (CAMPAS, 1992) and DHS 1992 (NPC, 1993). Some of these results will be summarized and discussed.

a) Comparison of Some Measures of Life Time Fertility

The data on current parities from EFS 1980, DHS 1988, PAP/CHILD 1991 and DHS 1992 surveys are cross-sectional and do not refer to the reproductive behavior of a cohort of women with growing age.

Therefore, current parity for younger women will reflect their fertility during a limited period only, while for older women this measure comes closer to their life time fertility.

The comparisons of the mean parities by age group for these surveys are shown in table (1). As expected, the parities increase rapidly with age since fertility levels have been high in the past in Egypt. On the average, the fertility increases by about one child for each five-year age group.

The parities for EFS show an increase compared to those for the DHS 1988, and PAP CHILD 1991 survey, starting for women aged 25. In fact this difference is not real because of the difference in time and different cohorts. On other hand, DHS 1988 survey has recorded comparatively lower parities (3.2) and this is theoretically not acceptable.

In fact, this comparison would be misleading because of the mean parities index is not very sensitive to changes (Juarez, 1983).

Another measure related to cumulative fertility is the mean children ever born to women aged 45-40, which can be taken as an indicator of the level of completed fertility.

As shown from table (1) there is a decrease in the measure of completed family size from 6.9, 6.3, and 6.0 for the year 1988, 1991, and 1992 respectively.

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Table (1)

Table (1): Mean Number of Children Ever Born
Per Ever Married Women By Current Age
1980 EFS, DHS 1988, and 1991 Egyptian PAP CHILD, and DHS 1992

Current Age	EFS 1980	DHS 1988	PAP CHILD 1991	DHS 1992
<20	0.63	0.5	0.75	.094
20-24	1.81	1.7	1.65	.928
25-29	3.07	2.9	2.88	2.406
30-34	4.61	4.0	3.95	3.725
35-39	5.79	5.3	4.88	4.712
40-44	4.46	5.9	5.48	5.485
45-49	6.87	6.3	5.96	6.005
ALL AGE	4.13	3.2	3.91	2.726

Source (1) CAPMAS, Egyptian Fertility Survey.1980 Vol.1. 1983.

Source (2) NPC, Demographic and Health Survey 1988, Cairo. 1989.

Source (3) CAPMAS. PAP CHILD Survey, Cairo. 1992.

Source (4) NPC, Demographic Health Survey 1992, Cairo. 1993.

b) Comparison of Some Measures of Current Fertility

In this section. two measures of current fertility are considered. These measures are the age specific fertility rates, and the total fertility rate(TFR). The procedure will be used for estimating ASFR's is relatively simple by classifying the number of births in the last five years by age of mother divided by the number of women-years lived in the same age group. These rates were multiplied by the proportion of ever married women from the household to adjust women who are not married at the time of the survey.

For the EFS (1980), DHS (1988), and PAP CHILD (1991), and DHS 1992 surveys, the results for the estimation of ASFR's are shown in table (2).

The pattern of age specific fertility rates begins with a relative minimum age around 15 years , then sweeps upward forming a broad. For the three surveys (1980-1991) the pattern is almost similar. The peak of the pattern also occurs for the age group 25-29, and this peak is usually reflected in a moderate change of fertility.

The level of current fertility is measured by the total fertility rates. The results from these surveys have shown a steady drop from 5.27, 4.66, 4.55 through to 3.90 respectively for the years of these surveys held in 1980, 1988, and 1992.

Table (2)
Age-Specific Fertility Rates for the
Five-Year Period to EFS 1980, DHS 1988,
1991 Egyptian PAP CHILD and the survey of DHS 1992

Age at Maternity	EFS 1980	DHS 1988	PAP CHILD 91	DHS 1992
15-19	099.30	083.00	076.0	063.0
20-24	255.50	237.00	223.0	208.0
25-29	285.20	215.00	251.0	222.0
30-34	217.40	198.00	188.0	155.0
35-39	130.50	120.00	112.0	89.0
40-44	048.20	044.00	044.0	43.0
45-49	015.50	005.00	015.0	6.0
TFR	5.27	4.66	4.55	3.9

Source: (1), (2), and (3), and (4) Ibid.,

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In short, the analysis of ASFR's is good enough to provide an idea of the levels and trends of fertility. However, it is limited in its capacity to reveal the smaller changes which may have occurred.

On the other hand, cumulated fertility of women by period and cohorts have similar disadvantages. Results can be difficult to interpret due to biases in the data.

5.2 Relational Gompertz Model Application

Figure (1) show the observed F points (current fertility) and P points (life time fertility) plotted against standard values i.e $Z(i)-e(i)$ and $g(i)$. The group average method was used for fitting the F points which was in 3 points (EFS 1980) 4 points for (DHS 1988), and 3 points for (PAP/child 1991) and DHS 1992. Points lies almost on a straight line is particularly from the middle to the last reproductive age groups. The plot of P points indicate a sharp trend this for age range. It might be the result of a combination of age errors, sampling errors (especially for and omission of children ever born).

The estimates of parameters α and β have been displayed in table (3). The estimates of the parameter α for reflects an early start of childbearing all surveys. The estimated β values show that fertility is constant for Egypt as a whole, in EFS 1980, and a sharp decline was found in DHS 1988, PAP/CHILD 1991, and DHS 1992 surveys.

Adjusted figures for the first age group for ASFR's have been obtained on multiplication by a factor 0.75 the second age group because it has been observed that many marriages still occur in two stages with initial engagement followed later by beginning of cohabitation. Women regard themselves as married between these stage and reported themselves as such in surveys (Hill, and Shorter 1984).

EFS 1980.

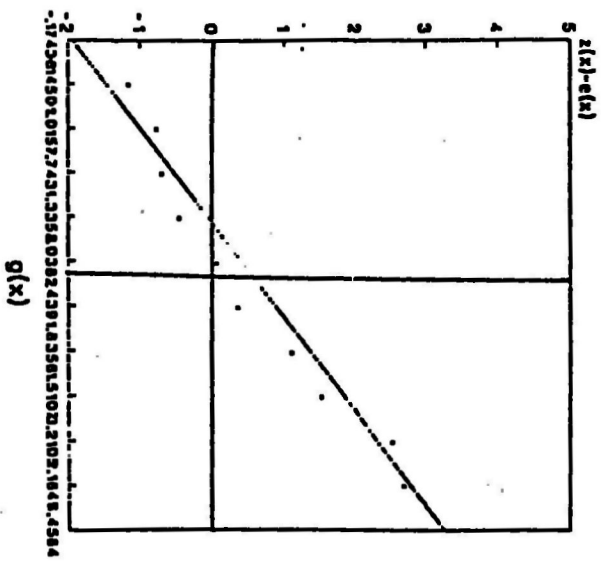
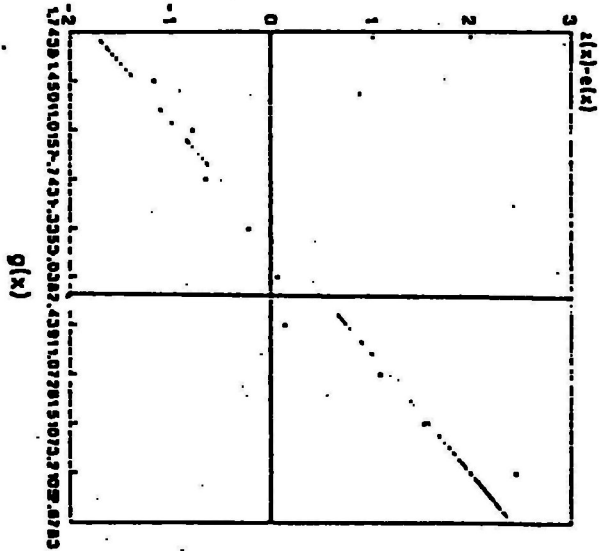
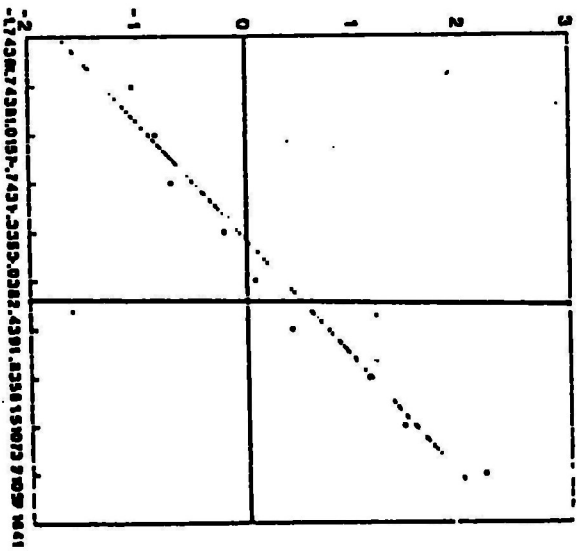


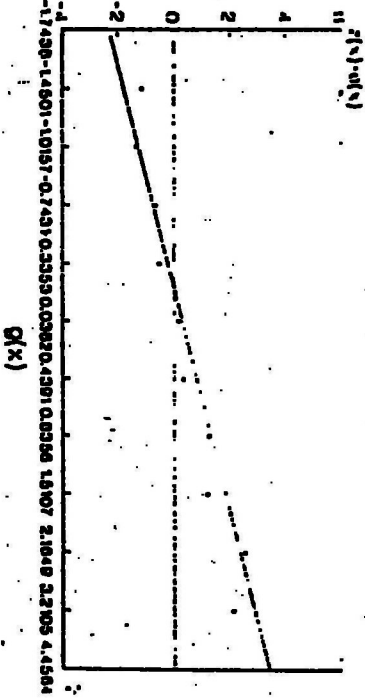
Figure (1) The Relational Gompertz
DHS 1988.



PAP 1991.



DHS 1992



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EFS 1980.

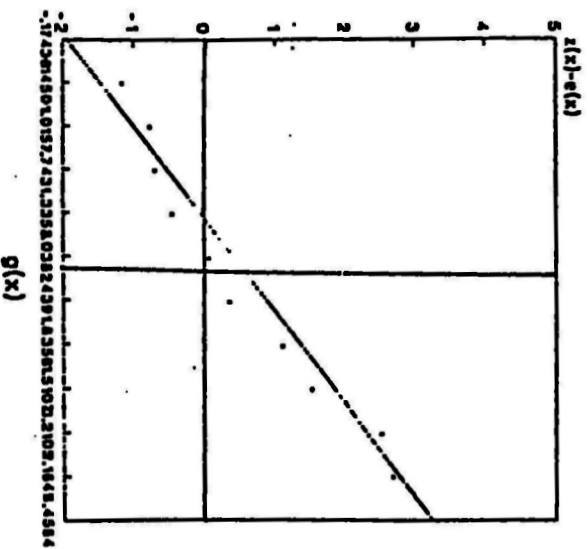
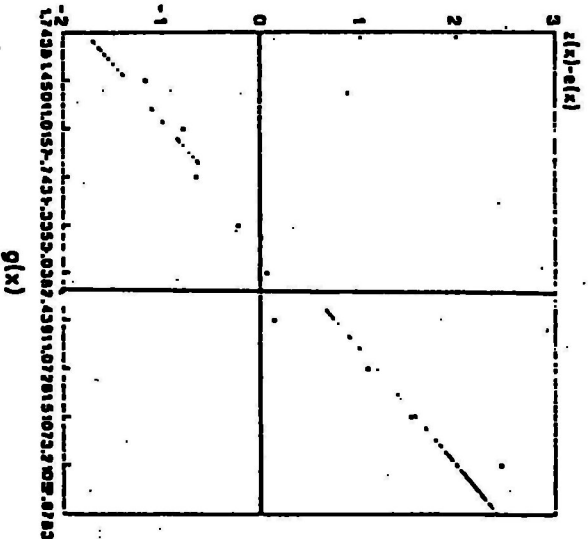
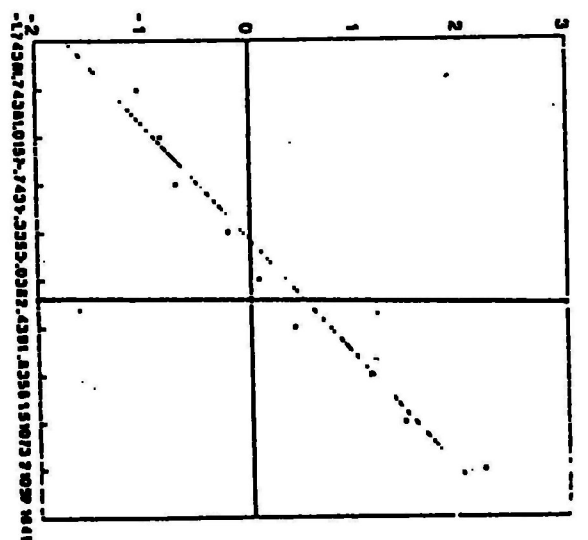


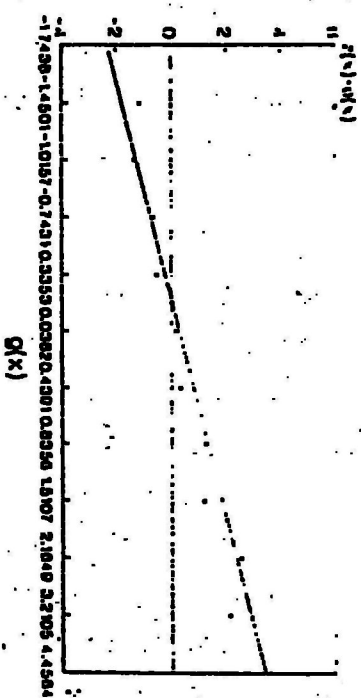
Figure (1) The Relational Gompertz
DHS 1988.



PAP 1991.



DHS 1992



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Using the estimates of the parameter α and β to data from mean annual births for the last five years to the standard mean parity values $Y(i)$, we obtain the total fertility rates. The estimates of total fertility show a steady fall with increasing age group of women, which could imply that the fertility pattern indicated by the reported $P(i)$ is different from that of the current rates. Another alternative explanation is that fertility has risen steadily or that births has been under-reported by women even in the younger age groups.

On the other hand, the estimates of α and β to the standard value of cumulative fertilities $y(x)$'s to obtain proportional cumulative fertility $F(x)$, decumulated and dividing by 5 yield the model age specific fertility rate $f(x)$. The reported ASFR's presented in table (4), shows that the peak of fertility is in the age group 20-24 for all areas. The age pattern of fertility is broadly similar in all surveys.

As shown at the bottom of table the estimated total fertility rates for all surveys are less than those from the observed total fertility rates calculated from birth history.

Two important points have emerged by an application of this technique. The first is that there is an under-reported births for all age groups, while the second, is that there is a very slight decline in fertility level in overall Egypt, during in the beginning of 1980's and a sharp decrease in 1988, as well as, in 1991, and DHS 1992.

Table (3)
Estimates of the Parameters of Gompertz Model
(α and β) for Various Regions in Egypt
EFS, 1980

Survey	Parameter	
	α = (location)	β = (spread)
EFS 1980	.096	.923
DHS 1988	.080	.913
PAP 1991	.142	.945
DHS 1992	-.004	1.096

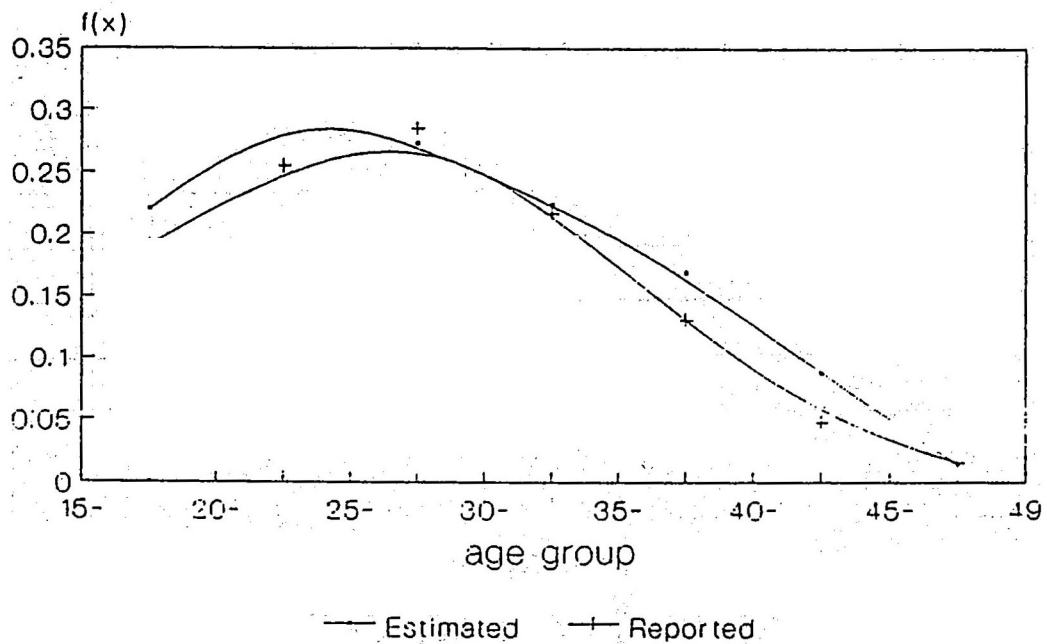
Table (4)
Estimated Age Specific Fertility Rates
by Using Relational Gompertz for
EFS 1980, DHS 1988, PAP 1991, and DHS 1992.

Age Group	EFS 1980	DHS 1988	PAP 1991	DHS 1992
15-19	220.00	194.20	213.6	099.0
20-24	303.40	268.50	287.8	222.72
25-29	273.00	243.90	252.1	215.3
30-34	223.70	202.10	201.2	165.3
35-39	168.90	154.60	147.4	107.4
40-44	086.60	080.80	072.8	042.6
45-49	013.80	013.30	011.0	004.4
T F R (est.)	006.51	005.16	05.04	4.28
T F R (obs.)	005.27	004.66	04.55	3.90

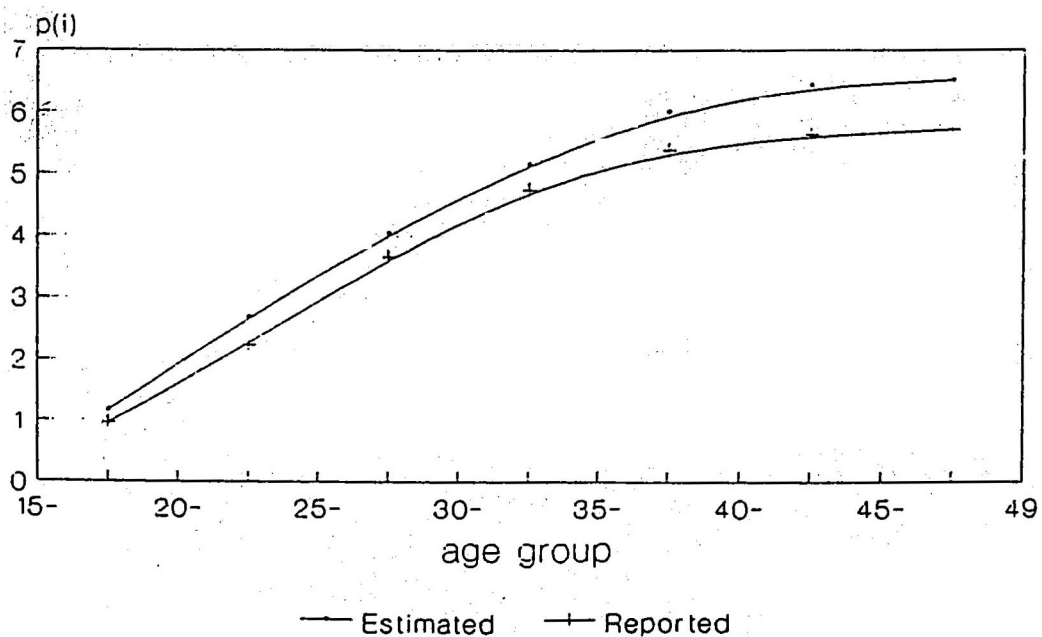
obs. = observed.

est. = estimated.

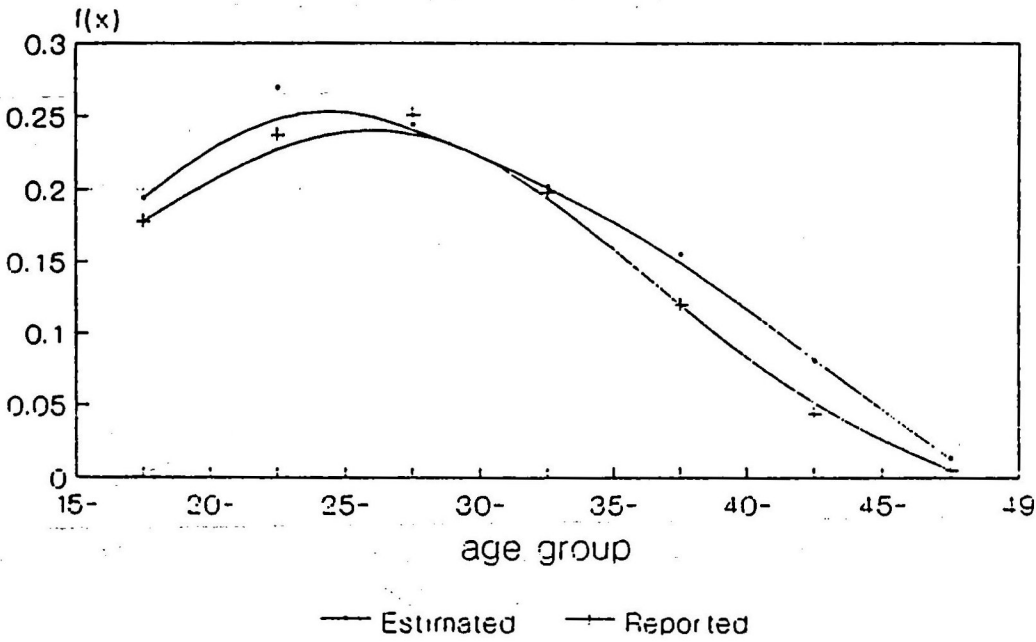
Fig(1) Estimated and Reported ASFR'S
Gompterz Model. EFS 1980



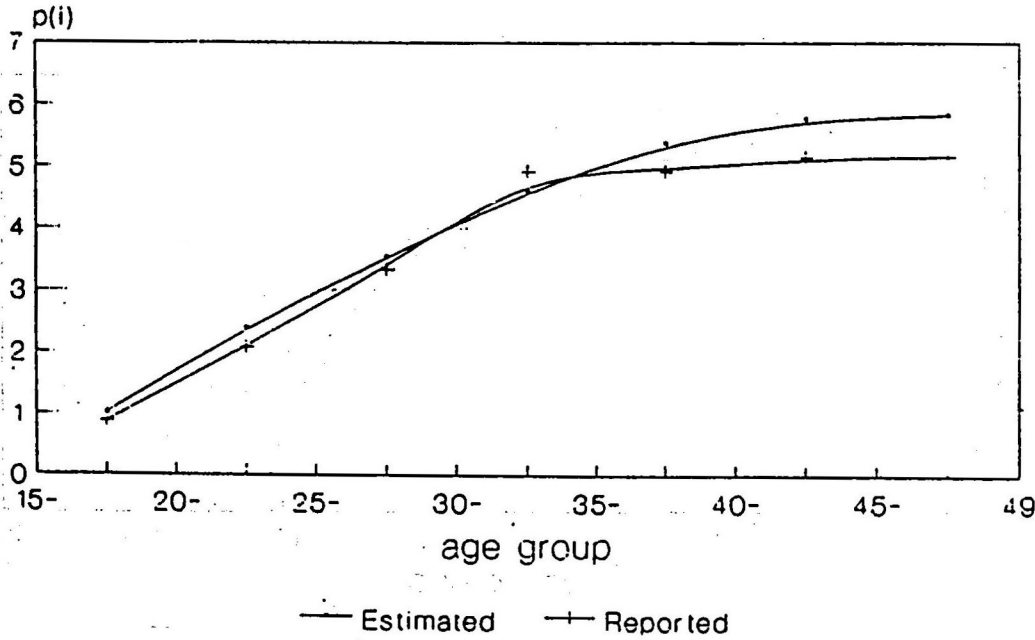
Fig(2) Estimated and Reported Parity
Gompterz Model. EFS 1980



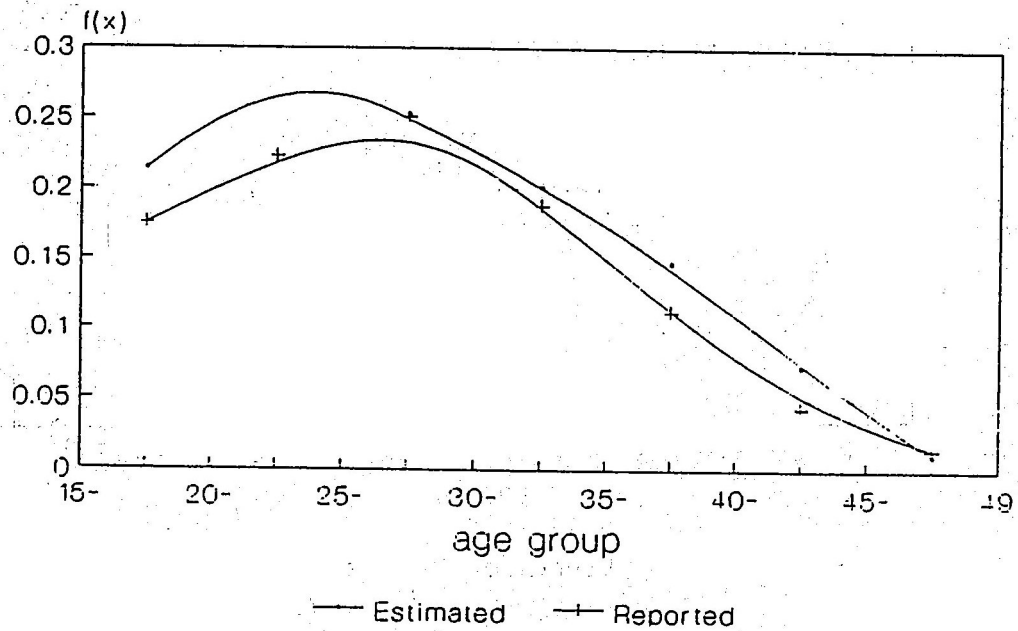
Fig(4.) Estimated and Reported ASFR'S
Gompterz Model. DHS 1988



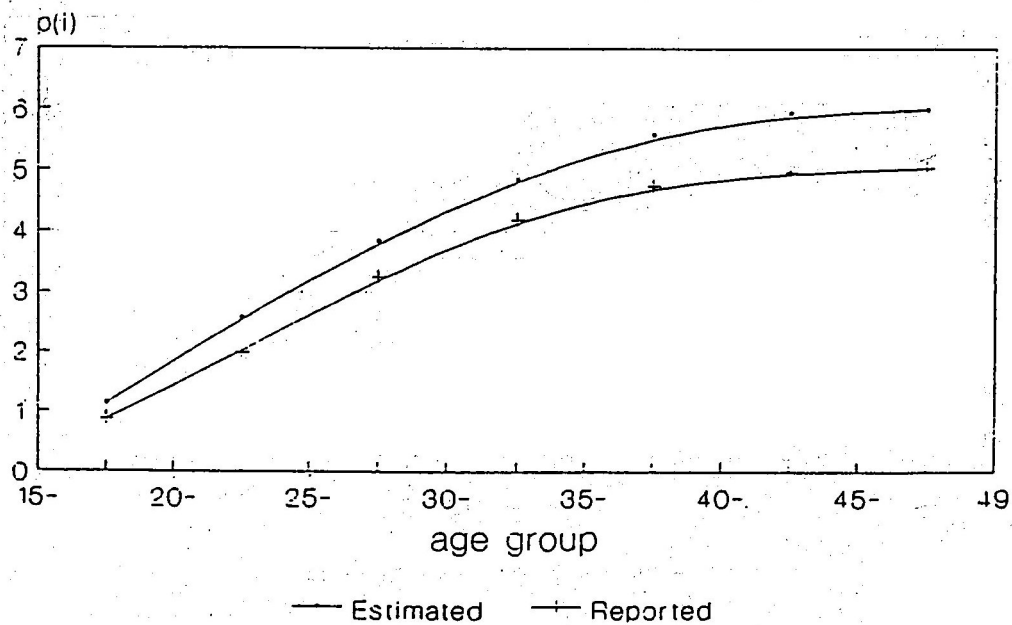
Fig(5) Estimated and Reported Parity
Gompterz Model. DHS 1988



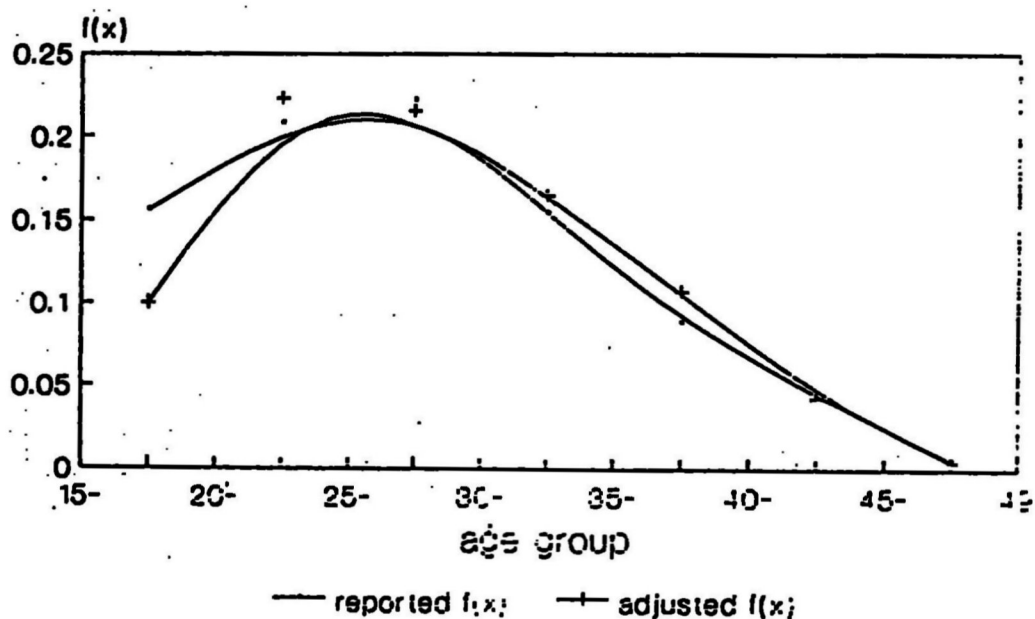
Fig(6) Estimated and Reported ASFR'S
Gompterz Model. PAP 1991



Fig(7) Estimated and Reported Parity
Gompterz Model. PAP 1991



Fig(8): Estimated and Reported ASFR'S.
Gompterz Model. DHS 1992



Fig(9): Estimated and Reported Parity.
Gompterz Model. DHS 1992

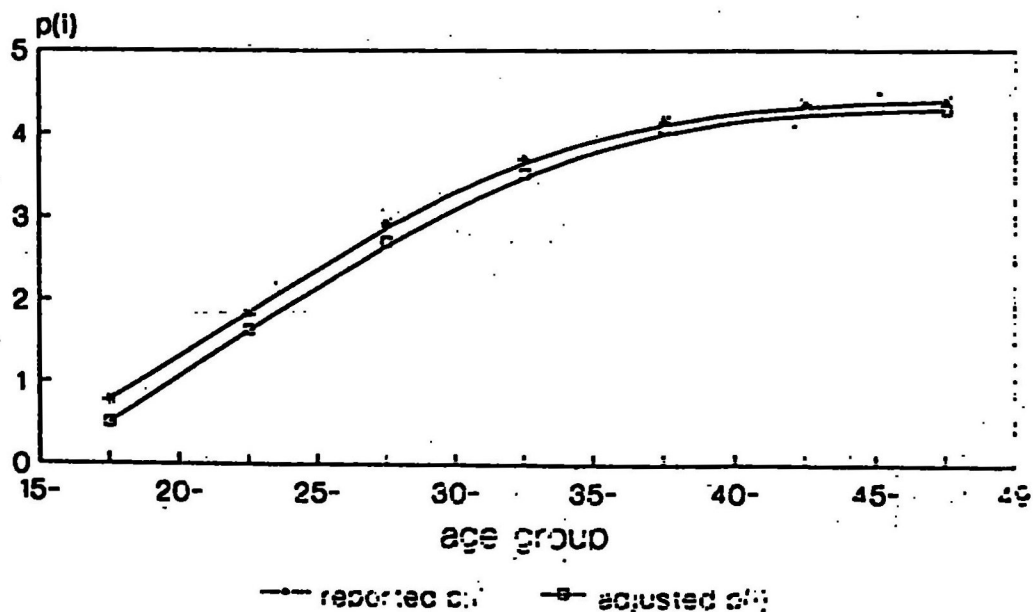


Fig (10) Comparison Estimated ASFR'S
for EFS80,DHS88,PAP91,and DHS92.

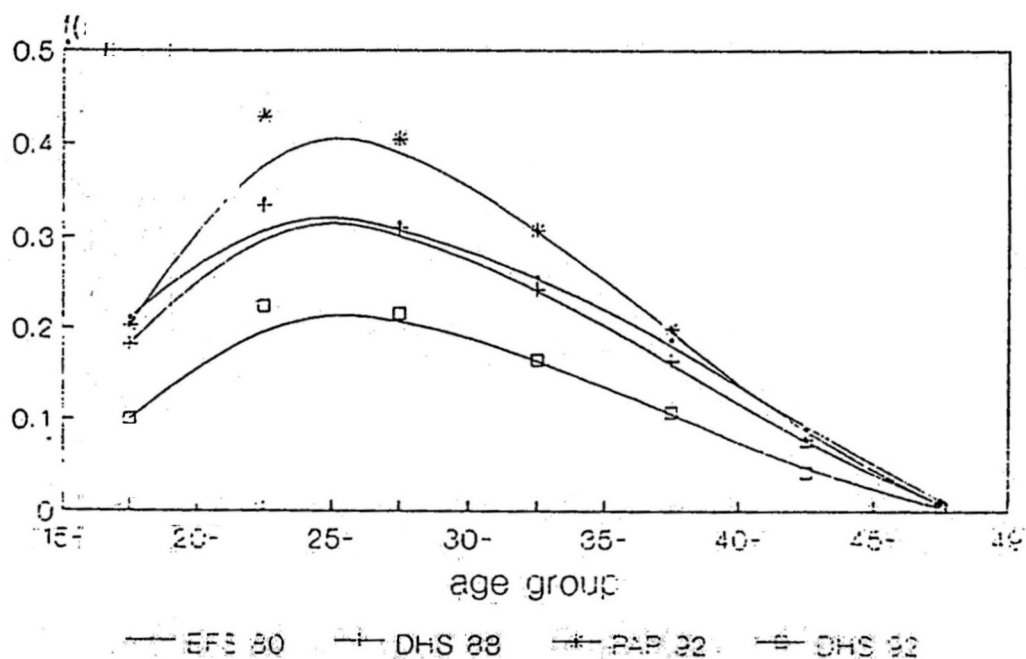
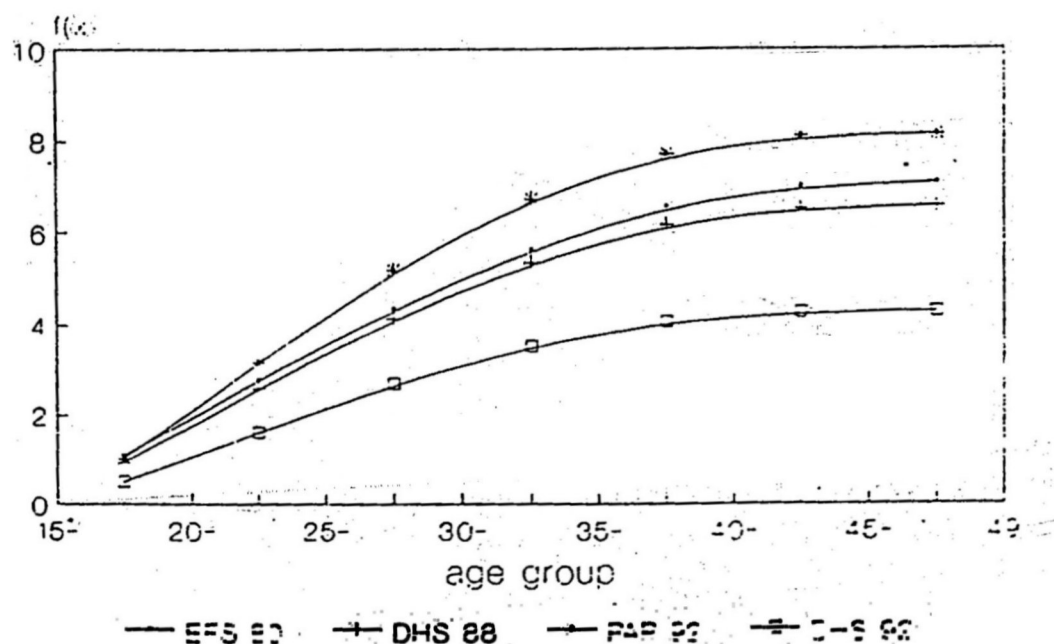


Fig (11) Comparison Estimated Parity
for EFS80,DHS88,PAP91,and DHS92.



6- Summary and Conclusions

It turns out that fertility is still high in Egypt although there is an indication that it has declined from the level prevailing in the recent past. The TFR consistently declined from 5.3 in 1980 through 1988 to 4.5 in 1991. This decline is inconsistent between the later surveys because there has been a sharp increase in the rate of contraceptive use, from 37.8% to 47.8 % .

Conventional measures, it is still difficult to show convincingly that a quantum fall in fertility has occurred. Results can be difficult to interpret due to particular biases in the data.

Also, indirect technique such as P/F ratios are not appropriate to use either, because the assumptions underlying them are justified. So, other techniques such as involving relational is needed to detect recent changes in fertility.

The Gompertz model has shown a real decline in fertility especially starting from 1988. The estimates for DHS 1988 are $\alpha = .913$ and $\beta = .080$, represent the slope and the intercept respectively. This is supported also by PAP/Child survey, 1991, and DHS 1992.

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Appendix
EFS (1980)

Table 1

Group	Age-specific fert. rates $f(x)$	Age X	Cumulated fertility $F(x)$	$Z(x)$ *	$e(x)$	$Z(x)-e(x)$	$g(x)$
15-19	0.1913	19.5	0.9565				
20-24	0.2155	24.5	2.2340	0.1646	1.3364	-1.1718	-1.450
25-29	0.2852	29.5	3.6600	0.7059	1.4184	-0.7125	-0.743
30-34	0.2174	34.5	4.7470	1.3469	1.2978	0.0491	-0.038
35-39	0.1305	39.5	5.3995	2.0495	0.9670	1.0825	0.835
40-44	0.0482	44.5	5.6405	3.1312	0.4509	2.6803	2.164
45-49	0.0155	49.5	5.7180	4.2943	0.0462	4.2481	4.456

$$* Z(x) = -\ln[-\ln \{F(x)/F(x+5)\}]$$

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Table 2

Age X	i	Mean Parities $P(i)$	$Z(i)$ *	$e(i)$	$Z(i)-e(i)$	$g(i)$
15	1	0.6300				
20	2	1.9100	-0.0539	1.2897	-1.3436	-1.7436
25	3	3.0000	0.6380	1.4252	-0.7872	-1.0157
30	4	4.6100	0.9000	1.3725	-0.4725	-0.3353
35	5	5.7900	1.4788	1.1421	0.3367	0.4391
40	6	6.4600	2.2119	0.7061	1.5058	1.5107
45	7	6.9700	2.7882	0.2763	2.5119	3.2105

$$* Z(i) = -\ln[-\ln \{P(i)/P(i+1)\}]$$

Parameters of the regression are : $\alpha = 0.09647$ $\beta = 0.92391$

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Table 3

i	Mean Parities P(i)	Zs(i)	Zhat(i) *	Phat(i) **	Fhat(i) ***
1	0.6300	-1.0787	-0.9002	0.0854	7.3740
2	1.8100	-0.3109	-0.1903	0.2981	6.0709
3	3.0700	0.3538	0.4234	0.5195	5.9093
4	4.6100	1.0569	1.0730	0.7004	6.4897
5	5.7900	1.9534	1.9013	0.8612	6.7229
6	6.4600	3.4130	3.2498	0.9620	6.7154
7	6.8700	6.0557	5.6915	0.9966	6.8932

* Zhat(i) = a + b * Zs(i)
 ** Phat(i) = exp(-exp(-Zhat(i)))
 *** Fhat(i) = P(i) / Phat(i)

Average of the first 5 values of Fhat(i) is F = 6.5134

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Table 4

Age group	Age X	Zs(x)	Zhat(x) *	Model Fhat(x) **	Model Age spec. fert. rates
-14	15	-1.75210	-1.5223	0.0666	0.0133
-19	20	-0.6400	-0.5422	1.1665	0.2200
-24	25	0.02564	0.1202	2.6834	0.3034
-29	30	0.70000	0.7432	4.0482	0.3730
-34	35	1.47872	1.4627	5.1667	0.2237
-39	40	2.62602	2.5227	6.1111	0.1689
-44	45	4.80970	4.5403	6.4442	0.0866
-49	50	*****	*****	6.5134	0.113

* Zhat(x) = a + b * Zs(x)
 ** Model fhat(x) = F * exp(-exp(-Zhat(x)))

Use arrows to scroll, Home for screen 1, End for screen 5, Esc to quit