A MODEL FOR COMPONENTS OF MARITAL FERTILITY

By

S.M. FARID

INTRODUCTION

The objective of this paper is to evaluate the role played by fertility and age composition of women at risk in annual fluctuation in the frequency of legitimate births.

The number of legitimate births in any given year is primarily determined by such factors as the quantity of fertility, the timing of births, the number of married women at the childbearling ages, their age distribution and their composition as to the duration of their marriages and the number of children prevously born. Thus, the frequency of births is influenced by a variety of factors of differing intensities operating with or against one another, and changes in initial size of successive birth cohorts throw no light on trends in these factors.

In this paper we shall employ the method of decomposition to evaluae the role played by certain demographic factors in annual changes in the frequency of legitimate births. The components model presented below decomposes the change in the number of legitimate births between two different dates into effcts of fertility, proportion of married women, size of the female population, age distribution of women at risk and interaction between fertility and age distribution. The model has been applied to England Wales marital fertility dato for the period 1956—70 (Farid, 1974).

The method of decomposition has been employed by several authors. For example, Kitagawa (195) used it to decompose changes in crude rates; Keyfitz (1968) used it to decompose changes in the mean age in the stable population into mortality, fertility & residual components; and Rethereford (1972) used it to examine the effects of tobacco smoking on the sex mortality diirerential.

NOTATION

The range of the reproductive span is taken as 15—44 years and all data are assumed to be presented in terms of standard five-year age groups.

B (t) : a vector (7 x 1) representing the age distribution of legitimate births at time t.

A (t) : a vector (7 x 1) representing age-specific fertility rates at time t.

N (t) : a diagonal matrix (7 x 7) showing the age distribution of married women at risk a time t. Its trace, tr N(t) gives the total number of married women at risk aged 15—44 at time t.

T (t) : a diagonal matrix (7 x 7) of the age distribution of all women at time t. Its trace gives the total female population in the age range 15—44 at time .

$E(t) = \frac{1}{r\ N\ (t)}$ N (t) : a diagonal matrix (7 x 7) of the percentage distribution of married women at time t.

p (t) = tr N (t) / tr T (t) : proportion of married women at ages 15—44 at time t.

THE PROBLEM

As already mentioned, our discussion will refer to the components of changes in the number and age distribution of legitimate births between two different dates. The age distribution of births at time t is given as.

$$B(t) = N(t).\ A(t). \qquad (1)$$

Thus, the vector of differences betwen the age distribution of births at ime t and that at time 0 may expressed as.

$$D = N(t).\ A(t) - N(O).\ A(O). \qquad (2)$$

Eq. (2) suggests an approach to the study of marital fertility patterns, that is changes in the number of births arise from three factors, viz.,

(a) changes in age-specific fertility rates;

(b) changes in the age distribution of women at risk : and

(c) interaction betweeen fertility and age composition.

Thus, the problem is to ascertain «how much of D is due to change in fertility schedule, how much to change in age composition of married women, and how much to interaction between fertility and age compos-ition?»

THE FERTILITY COMPONENT

The measurement of the effect of changes in age-specific fertility rates between two different dates on the age distribution of births, may be based on the technique of direct standardization. Thus, the product of the matrices n (O) and A (t) would yield the age distribution of births which would have occurred if the population of married women at time t had the same age distribution as the base population while retaining its observed fertility schedule.

The vector (3)

$$D\ (1) = N(0).\ \left[A(t) - A(0)\right], \qquad (1)$$

would thus indicate how much of D is due to changes in age-specific fertility rates independent of changes in age composition.

THE AGE COMPOSITION COMPONENT

The measurement of the effects of changes in the age distribution of the population of married women on the age distribution of births may be done by employing the technique of indirect standardization. The multiplication of matrix N (t) by vector A (O) would result in a diagonal matrix showing the age distribution of births which would have occurred at time t with its age composition, if its fertility schedule had been exactly the same as for the base population.

The difference between the product N (t), A (O) and the observed births at time O, would indicate how much of D is due to changes in the age composition of the population of married women independent of changes in the fertility schedule.

The age composition component may thus be written as

$$D(2) = \left[N(t) - N(0)\right] \cdot A(0) \cdot \qquad (4)$$

The change in the age composition of the population of women at risk as measured in Eq. (4) by the difference [N (t) - N (O)] - may be considered to have been caused by one or more of the following factors :

(i) a change in the proportion of married women;

(ii) a change in the size of the female population at the reproductivespan ; and

(iii) changes in the proportional age distribution of the population of married women.

Thus, the age composition component may be further divided into effects of the three factors mentioned adove.

(i) Effect of Changing Proportion Married.

The effect of a change in the level of nuptiality may be measured by multiplying the proportion of married women at time O by the total female population at time t, o yield the expected number of married women at time t, if both the base and the given populations did have the same level of nuptiality.

Multiplication of the product p (O). tr T (t) into each of the elements of marix E (t) would give the expected age distribution of married women at time t,

$$p(0) \cdot T(t) \cdot E(t) = \frac{p(0)}{p(t)} \cdot N(t) \cdot \qquad (5)$$

The multiplication of matrix $\frac{p\ (0)}{p\ \ (t)}$ N (t) by vector A (O) would result in a diagonal matrix showing the age distribution of births which would have occurred at the given time period t, if its fertlity schedule and level of nuptiality had been the same as those for the base time period,

$$\frac{p(0)}{p(t)} .\ N(t).\ A(0). \qquad (6)$$

Subtracting (6) from the product N (t). A (O) gives that part of the age composition component which is due to a change in the level of nuptiality,

$$D\ (2.1) = \left[1 = \frac{p(0)}{p(t)}\right].\ N(t).\ A(0). \qquad (7)$$

(ii) Effect of changing total female population.

The effect of a change in the size of the female population may be measured by assuming tha the level of nuptiality and total female popultion were the same in the given population as in the base population, i.e. the given population contained the same number of married women as did the base population.

The multiplication of tr N (O) - the total number of married women at time O-into each of the elements of matrix E (t) would thus result in the expected age distribution of married women at under the above mentioned assumption,

$$tr\ N(0).\ E(t) = \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)} .\ N(t). \qquad (8)$$

The expected age distribution of births of the hypothetical population given in Eq. (8) at time t, if is fertility schedule were the same as for time O, is given as.

$$\frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)} .\ N(t).\ A(0). \qquad (9)$$

Thus, the part of the age composition component which is due to a change in the size of the female population may be expressed as

$$D(2.2) = \frac{p(0)}{p(t)} .\ N(t).\ A(0) - \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)} .\ N(t).\ A(0)\ . \qquad (10)$$

(iii) Effect of changing age distribution of married women.

The effect of changes in the age distribution of married women, on the age distribution of births, may be measured by subtracting the births at time O from the births calculated in Eq. (9).

$$D\ (2.3) = \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)}.\ N(t).\ A(0) - N(0).\ A(0). \qquad (11)$$

The second left-hand term in Eq. (11) may be rewritten as

$$\begin{aligned} N(0).\ A(0) &= tr\ N(0).\ E(0).\ A(0) \\ &= tr\ N(0).\ E(t).\ E^{-1}(t).\ E(0).\ A(0) \\ &= \frac{tr\ N(0)}{tr\ N(t)}.\ N(t).\left[E^{-1}(t).\ E(0)\right].\ A(0) \qquad (12) \\ &= \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)}.\ N(t).\left[E^{-1}(t).\ E(0)\right].\ A(0). \end{aligned}$$

Thus, Eq. (11) may be rewritten as

$$D(2.3) = \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)}.\ N\ (t).\ A(0) - \frac{p(0).\ tr\ T(0)}{p(t).\ tr\ T(t)}.\ N(t).\left[E^{-1}(t).\ E(0)\right].\ A(0). \qquad (13)$$

THE INTERACTION COMPONENT

The intreaction between changes in age-specific fertility rates and changes in age-composition of women at risk may be measured by subtracting the sum of the fertility components and the three age-composition components from the total change. The interaction term is thus given as

$$D(3) = \left[N(t) - N(0)\right].\left[A(t) - A(0)\right]. \qquad (14)$$

THE COMPLETE FORMULA

The complete components formula may now be expressed by the following identity.

$$D = D(1) + D(2.1) + D(2.2) + D(2.3) + D(3)$$

$$\begin{aligned}
N(t).\,A(t) - N(0).\,A(0) &= N(0).\left[A(t) - A(0)\right] \\
&+ \left[1 - \frac{p(0)}{p(t)}\right].\,N(t).\,A(0) \\
&+ \frac{p(0)}{p(t)}\left[1 - \frac{tr\ T(0)}{tr\ T(t)}\right].\,N(t).\,A(0) \\
&+ \frac{p(0).\,tr\ T(0)}{p(t).\,tr\ T(t)}.\,N(t).\left[I - E(t)^{-1}E(t)\right].\,A(0) \\
&+ \left[N(t) - N(0)\right].\left[A(t) - A(0)\right],
\end{aligned} \qquad (15)$$

where I denotes an identity martix.

Putting

$$K = 1 - \frac{p(0)}{p(t)}$$

$$r = 1 - \frac{tr\ T(0)}{tr\ T(t)}$$

Then, the components identity becomes.

$$\begin{aligned}
N(t).\,A(t).\,- N(0).\,A(0) &= N(0).\left[A(t) - A(0)\right] \\
&+ K.\,N(t).\,A(0) \\
&+ (1 - K).\,r.\,N(t).\,A(0) \\
&+ (1 - K).\,(1 - r).\,N(t).\left[I - E(t)^{-1}E(t)\right].\,A(0) \\
&+ \left[N(t) - N(0)\right].\left[A(t) - A(0)\right].
\end{aligned} \qquad (16)$$

Thus, the change in the number of legitimate births between two diwerent dates is decomposed into five components :

(i) D (1) measures the contribution of changes in age-specific fertility rates.

(ii) D (2.1) measures that part of the total change which is due to a change in the proportion of married women at the childbearing ages.

(iii) D (2.2) measures the contribution of the change in the size of the female population at the reproductive span.

(iv) D (2.3) measures the contribution of the change in the degree to which married women are concentrated at different ages.

(v) The fifth component, which is a residual term, may be interpreted mathematically as the contribution of the interaction between fertility and age composition. The interaction is described by the difference between the effect of changing fertility at time t level of age composition and the effect of changing fertility at time O level of age composition (Keyfitz, 1968).

REFERENCES

1. FARID, S. M. (1974) : Components of Period Fertility in England and Wales, *Journal of Biosocial Science.* 6 : 59—66.
2. KEYFITZ, N. (1968) : Changing Vital Rates and Age Composition. *Population Studies.* 22 : 235—251.
3. KITAGAWA, E. (1955.) : Components of a Difference Between Two Rates. *Journal of the American Statistical Association* 50 : 1168—1194.
4. RETHERFORD, R. D. (1972) : Tobacco Smoking and the Sex Mortality. Differential. *Demography.* 9 : 203—216.