A PARITY-SPECIFIC FERTILITY TABLE FOR EG
APPLICATION OF NOUR'S METHOD

Amelia Dale Horne

1.0 Introduction

Fertility tables have long been used to summarize the reproductive behavior of a population, and , as with life-tables of mortality, have used women's age as the important variable relating to their fertility performance. In the present paper, a method of constructing a fertility table based on the observed current parity distribution, developed by Nour (1984), is used to obtain a parity-specific fertility for Egypt . Data on the parity-distribution of Egyptian women were obtained from the 1980 Egyptian Fertility Survey (EFS), (CAPMAS,1983a).

Nour's method is based on the assumption that women at a given parity and age will have the same fertility behavior regardless of their previous experience. His fertility table represents a departure from the tradition of considering a woman's age, closely related to her fecundability, as the underlying variable for the study of reproduction. Chiang and Van Den Berg (1982) point out that the advent of modern contraceptive methods has made the reproductive process more a matter of a couple's choice than a function of a woman's fecundability.For this reason they propose the use of womwn's parity as the basis for summarizing the reproductive experience of a population. From a policymaking point of view, particularly regarding family planning policy, a parity-specific fertility table may be more practical the usual age-specific type, since in most developing societies a woman's parity is more easily determined than her age. This is especially true if parity is considered to be effectively the number of living children.

Nour's statistical model for the construction of a parity-specific fertility table is given in the next section. Because of the complexity of the model, details are given both of the statistical model and of the method of constructing the table, the latter of which are given in section 3.0 and are not available in Nour's paper.

2.0 Nour's Model

The following development of the statistical model and the notation depend upon the paper by Nour (1984).

Consider the childbearing age interval $X = [x_0, x_\omega]$ and let $\{I(X); x \in X\}$ be the parity of a woman at age $x \in X$. Assume that $\{I(x); x \in X\}$ is a Markov process where for every $x \in X$, $I(x)$ has a finite space $\{S = 0,1,\ldots,k\}$. With K being the maximum attainable parity. The Markovian assumption suggests that all women who have a given parity at the same age will have the same probabilistic behavior regardless of their past experiences. Underlying the process is a set of instantaneous fertility rates $\{r_0(x), r_1(x), \ldots, r_{k-1}(x)\}$ such that

$r_i(x)\Delta x + 0(\Delta x) = p$ {a women of age x and parity i will have a live birth in the age interval $(x, x+\Delta x)$,} $x \in X$ and $i \in S$.

The rates $r_i(x)$ satisfy the relationship

$$r_i(x) = \lim_{\Delta x \to 0} \frac{1 - q_{i,i}(x, x+\Delta x)}{\Delta x}, \qquad (1)$$

Where $q_{i,i}(y,x) = P\{I(x) = i \mid I(y) = i\}$, $i \in S$, $y \leq x$. The unique solution to (1), subject to the initial condition $q_{i,i}(x,x) = 1$, is $q_{i,i}(y,x) = \exp\{-\int_y^r r_i(t)\,dt\}$. (2)

The function $P_i(y) = 1 - q_{i,i}(y, x_\omega)$, where x_ω is the maximum age of childbearing, may be termed the age-specific parity progression ratio. Let $g_i(y)$, $y \in X$ and $i \in S$, be the age distribution of women whose current parity is i. The usual parity progression ratios, denoted by p_i, are computed as

$$p_i = E\{p_i(y)\} = \int p_i(y) \cdot g_i(y)\,dy. \qquad (3)$$

The synthetic cohort estimates of the distribution of final parity may be obtained from p_i by defining the parity progression ratio in the context of cohort analysis as

$$P_i = P\{ N \geq i \mid N \geq i \},$$

where N denotes final parity then the probability disribution of N is given by

$$p\{N = i\} = (1-p_i) \prod_{j=0}^{i-1} p_j \qquad (4)$$

Let y_i be the waiting time until the birth of the birth of the (i+1) st child and Z_i be the waiting time until completion of childbearing for a woman whose current parity is i. Also, the indicator random variable $_i\delta$ is defined by

$$\delta_i = \begin{cases} 1 & \text{if a woman whose current parity is } i \text{ goes on to have an } (i+1) \text{ st child,} \\ 0 & \text{otherwise :} \end{cases}$$

Then k-1

$$Z_i = \sum_{j=i}^{k-1} \xi_j^i \, V_j,$$

Where

$$\beta_j^i = \prod_{t=i}^{j} \delta_t, \qquad i,j \in S. \qquad (5)$$

Also,

$$E(Z_i) = \sum_{j=i}^{k-1} p\{\beta_j^i = 1\}.\ E\{V_j | \beta_j^i = 1\}$$

$$= \sum_{j=i}^{k-1} p\{\beta_j^i = 1\}\cdot\ E\{V_j | \delta_j = 1\},$$

$$= \sum_{j=i}^{k-1} p\{\beta_j^i = 1\}.\ E\{U_j\}, \qquad (6)$$

Where U_j is the waiting time until the (j+1)st birth for a woman whose current parity is j and who goes on to have a (j+1)st child. Also

$$p\{\beta_j^i = 1\} = p\{\delta_t = 1 \text{ for all } i \leq t \leq j\}$$

$$= \prod_{t=i}^{j} p_t - A_j^i, \qquad (7)$$

Where p_t, $t \in S$, is the parity progression ratio defined in (3).

Therefore

$$E(Z_i) = \sum_{j=i}^{K-1} A_j^i E(U_j). \qquad (8)$$

To evaluate $E(U_i)$, not that

$$E(U_i) = E_{Y_i}(E_{X_{i+1}}(X_{i+1} - Y_i | Y_i = y, I(y) = i, X_{i+1} < X_\omega)), \qquad (9)$$

Where X_{i+1} is the age at which a woman has her (i+1)st birth and Y_i is the current age at parity i. Since

$$P\{X_{i+1} \geq x [Y_i = y, I(y) = i\} = q_{i,i}(y,x),$$

Where $q_{i,i}(y,x)$ is given by (2), then

$$p\{X_{i+1} > x \mid Y_i = y,\ I(y) = i,\ X_{i+1} < x_\omega\} = \begin{cases} 1, & x \le y, \\ \dfrac{\exp\{-\int_y^x r_i(t)dt\} - \exp\{-\int_y^{x_\omega} r_i(t)dt\}}{1 - \exp\{-\int_y^{x_\omega} r_i(t)\,dt\}}, & y < x \le x_\omega \\ 0, & x < x_\omega . \end{cases} \quad (10)$$

Therefore,

$$E_{X_{i+1}}(X_{i+1} - Y_i \mid Y_i = y,\ I(y) = i,\ X_{i+1} < x_\omega)$$

$$= \int_0^\infty p\{X_{i+1} > X \mid Y_i = y,\ I(y) = i,\ X_{i+1} < x_\omega\}dx - y$$

$$= \frac{\int_y^{x_\omega} \exp\{-\int_y^x r_i(t)dt\}dx - (x_\omega - y)\exp\{\int_y^{x_\omega} r_i(t)dt\}dx - y}{1 - \exp\{-\int_y^{x_\omega} r_i(t)dt\}} \quad (11)$$

By taking expectation with respect to y_i on both sides of (11), the required $E(U_i)$ as defined by (9) is obtained.

The functions p_i and $E(U_i)$ may be computed by approximating $r_i(x)$ by a step function. The reproductive interval $X = [x_0, x_\omega]$ is divided into n disjoint age groups of the form $(a_j, a_{j+1}]$, where $a_1 = x_0$ and $a_{n+1} = x_\omega$, so that $r_i(X)$ is approximately constant within each of these age groups. That is

$$r_i(x) = r_{ij}, \qquad a_j < x \le a_{j+1} . \quad (12)$$

Then

$$P_i = 1 - \sum_{j=1}^{n} B_{ij} \cdot g_i(j) , \qquad (13)$$

and

$$E(U_i) = \sum_{j=1}^{n} \frac{C_{ij} - (x_\omega - y_j^*)B_{ij}}{1 - B_{ij}} \cdot g_i(j), \qquad (14)$$

Where $g_i(j) = P\{a_j < Y_i \leq a_j+_1\}$, $j = 1,\ldots,n$; y_j^* is the midpoint of the age interval $a_j < y \leq a_j+_1$, representing the age at i-th parity, Y_{ij} and

$$B_{ij} = \exp\{-(a_{j+1} - y_j^*)r_{ij} - \sum_{k=j+1}^{n} (a_{k+1} - a_k)r_{ik}\} \qquad (15)$$

and

$$C_{ij} = \frac{1}{r_{ij}} \{1 - \exp\{-r_{ij}(a_{j+1} - y_j^*)\}\}$$

$$+ \exp\{-r_{ij}(a_{j+1} - y_j^*)\}\left\{\sum_{k=j+1}^{n}\left[\frac{1}{r_{ik}}\{1 - \exp\{-r_{ik}(a_{k+1} - a_K)\}\} \cdot \exp\{-\sum_{u=1}^{k-j-1} r_{i(k-1)}(a_{j+u+1} - a_{j+u})\}\right]\right\} \qquad (16)$$

3.0 Application of Nour's Model to the EFS Data

The fertility table prescribed by Nour's model is completely specified in terms of the parameters p_i and $E(U_i)$.. Thus, it is helpful to see first how these intermediate figures are obtained.

Table 1 gives illustrative calculations to obtain the parity progression ratio, p_i, for parity 0. These computations were repeated for each parity group.

The fertility rates for parity i and age group j, the r_{ij}'s in column (3), are defined as

$$r_{ij\,(\text{year } t)} = \frac{\text{number of births of order } (i+1) \text{ to women of parity } i \text{ and age group } j \text{ during } t}{\text{number of women of parity } i \text{ and age } j \text{ during } t} \qquad (17)$$

TABLE 1

ILLUSTRATIVE CALCULATION OF PARITY PROGRESSION RATIO FOR PARTIY 0

EGYPT, 1980

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	
Parity i i	Age Group j	Fertility[a] Rate r_{ij}	y_j^*	$a_{j+1}-y_j^*$	-(3)x(5)	$5xr_{ik}$ k=j+1	(6)-(7)	B_{ij}[b] $=e^{(8)}$	$g_i(j)$[c]	(9)x(10)	parity[d] progression Ratio i - (11)
0	1	.331224	17.5	2.5	- .82806	5.8126	-6.64066	.001306	.3666	.000479	
	2	.495726	22.5	2.5	-1.239315	3.33397	-4.573285	.0 0324	.2976	.003072	
	3	.375796	27.5	2.5	- .93949	1.45499	-2.39448	.091220	.1493	.013619	
	4	.184615	32.5	2.5	- .461538	0.531915	- .993453	.370296	.0639	.023662	
	5	.106383	37.5	2.5	- .265958	0	- .265958	.766472	.0443	.033955	
	6	0	42.5	2.5	-	0	0	1	.0443	.0443	
	7	0	47.5	2.5		0	0	1	.0340	.0340	
										=.153087	.846913

[a] Calculated by definition (17).
[b] Calculated by formula (15).
[c] Calculated from table (2).
[d] Calculated by formula (13).

Source: 1980 Egyptian Fertility Survey.

The numerator is given as the number of births in the preceding twelve months, and the denominator as the number of women at exactly six months prior to the survey date.

The B_{ij}'s in colum (9) are the probabilities of not having a birth in the remaining age intervals. These probabilities are computed according to formula (15).

The $g_i(j)$'s in column (10) give the age distribution of women for parity 0. Alternatively, $g_i(j)$ may be viewed as the probability that a woman's age at the i-th parity is greater than a_j and less than or equal to a_{j+1}. These probabilities were obtained from the percentage distribution of women by current parity and age, presented in table 2.

The parity progression ratio, calculated by formula (13), is 1 minus the summation of column (11). This ratio, p_i, gives the probability that final parity is greater than i, given that it is at least i.

The other essential quantity of the fertility table is the $E(U_i)$, or the expected waiting time until the (i+1)st birth for a women with current parity i. $E(U_i)$ is easily attained by formula (14) once the intermediate figure C_{ij} is obtained. The calculation of the calculation of the C_{ij}'s was done by formula (16), and is illustrated for parity 0 in table 3. These calculations were repeated for each parity group.

4.0 <u>Besults</u> Table 4 give the parity-specific fertility table for Egypt, based on the EFS data. Because of relatively small numbers of births of order i+1 in the parity groups of seven or more children, these groups were combined.

TABLE 2

PERGENTAGE DISTRIBUTION OF WOMEN BY CURRENT AGE AND PARITY
EGYPT, 1980

Current	Parity								
Age	0	1	2	3	4	5	6	7+	Tatal
< 20	36.66	22.69	4.92	1.50	0.53	0.12	0.00	0.00	7.72
20-24	29.76	38.80	37.00	28.25	14.66	3.02	0.66	0.25	18.18
25-29	14.93	20.19	28.69	32.46	31.22	21.71	15.66	3.48	19.30
30-34	6.39	7.78	14.30	18.05	24.58	30.88	27.76	16.10	17.33
35-39	4.43	4.72	7.06	10.85	14.03	19.42	25.26	27.53	15.12
40-44	4.43	2.41	4.65	5.80	8.12	14.60	18.68	26.74	12.07
45-49	3.40	3.43	3.40	3.09	6.86	10.25	11.97	25.89	10.28
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Source: Egyptian Fertility Survey, 1980.

TABLE 3.

ILLUSTRATIVE CALCULATION OF C_{ij} POR PARITY 0

EGYPT, 1980

(1)	(2)	(3)	(4)	(5)	(6)								(7)	(8)
Parity i	Current Age Group j	$-r_{ij} x 2.5$	$e^{(3)}$	1-(4)	$\frac{(5)}{r_{ij}}$	$(7)=\sum_{k=j+1}^{n}\left[\frac{1}{r_{ik}}\{1-\exp\{-r_{ik}(a_{k+1}-a_k)\}\}\exp\{-\sum_{u=1}^{k-j-1} r_{i(k-1)}(a_{j+u+1}-a_{j+u})\}\right]$							$\sum_k$	C_{ij} [a] =(6)+(4)x(7)
						k = 2	3	4	5	6	7			
0	1	-.82806	.4369	.5631	1.7001	1.8481	.1891	.0762	.2432	-	-		2.3565	2.7296
	2	-1.23932	.2896	.7104	1.4331	-	2.2546	.4987	.6121	-	-		3.3653	2.4076
	3	-.93949	.3908	.6092	1.6210	-	-	3.2647	1.5406	-	-		4.8052	3.4990
	4	-.46154	.6303	.3697	2.0025	-	-	-	3.8777	-	-		3.8777	4.4466
	5	-.26596	.7665	.2335	2.1952	-	-	-	-	-	-		-	-
	6	0	1	0	-	-	-	-	-	-	-		-	-
	7	0	1	0	-	-	-	-	-	-	-		-	-

[a] Calculated by formula (16).

Soure: Egyptian Fertility Survey data.

TABLE 4

PARITY - SPECIFIC FERTILITY TABLE

Parity i	Mean age of Women at parity i $\bar{Y}_i$	Parity progression Ratio P_i	e_i	$E(U_i)$ in Years	L_i	T_i	e_i in Years	Mean age at Last Birth in Years $\bar{A}_L$
0	24.06	.84691	10.000	1.9086	16,164	75,910	7.59	31.65
1	24.96	.82579	8,469	1.5185	10,620	59,746	7.05	32.01
2	27.51	.81166	6,994	1.6898	9,593	49,126	7.02	34.53
3	29.19	.81821	5,677	4.8651	22,598	39,533	6.96	36.15
4	31.62	.71537	4,645	2.0205	6,714	16,935	3.65	35.27
5	34.69	.59183	3.323	3.5475	6,978	10,221	3.08	37.77
6	36.13	.56392	1,967	2.9243	3,243	3,243	1.65	37.78
7	39.87	-	1,109	-	-	-	-	-

Source: 1980 Egyptian Fertility Survey data.

The quantity $\bar{y}_i$ in table 4 is the observed mean age of the women who are currently at parity i.

The column of ℓ_i's are the numbers of women out of ℓ_0 (10,000) who are expected to have a final parity equal to or greater than i, and

$$\ell_{i+1} = p_i \ell_i . \tag{18}$$

The total exposure time to childbearing of women of parity i is given by L_i, Which is calculated as

$$L_i = \ell_{i+1} E(U_i) . \tag{19}$$

The column of T_i's gives the total reproductive spa span remaining after the i-th birth, and is computed as

$$T_i = \sum_{i=j}^{K-1} L_i . \tag{20}$$

The quantity e_i is the expected waiting time until the last birth for a woman of current parity i. It is computed as

$$e_i = T_i/\ell_i . \tag{21}$$

Finally, the mean age at the birth of the last ch child is obtained as

$$\bar{A}_L = \bar{y}_i + e_i . \tag{22}$$

One can see from the parity progression ratios in table 4 that women who currently have no children have a probability of .85 of having at least one child Symptomatic of a highfertility environment, women with as many as six children still have a probability as great as. 56 of hoving more children.

The expected waiting times to the next birth for women of each parity, the $E(U_i)$'s, indicate close spacing of early births (less than two years), with increased spacing at later parities.

The mean age at last birth, which is a projected age, in Egypt exhibits a ageneral tendency to increase with parity. The exception is that women with currently three children are expected to end childbearing about one year later than women who presently have four children. This difference is because of the relatively long waiting time (4.865 Years) expected between the third and fourth births. A possible explanation is that many Egyptian women use contraception after the third child. previous published results from the EFS data indicate that "three living children [and] age 28 ... are the effective points at which more than 50 percent of [Egyptian] women wish to limit their family size" (CAPMAS, 1983b, p.93).

Egyptian women who currently have no children are expected to end childbearing at just under 32 years of age. Women currently with six children are expected to have their last chiled at almost 38 gears of age.

Such are the prospeets for a new cohort of Egyptian women, should they experience the family building process as indicated by the EFS data.

REFERENCES

CAPMAS. 1983a. Egyptian Fertility Survey, 1980. Standard Recode, Version 3. Cairo: Central Agency for Public Mobilization and Statistics.

CAPMAS. 1983b. Egyptian Fertility Survey, 1980. Volume 2: Fertility and Family planning. Cairo: Central Agency for pufilic Mobilization and Statistics.

Chiang, Chin Long and Bea J. Van Den Berg. 1982. "A Fertility Table for the Analysis of Human Reproduction." Mathematical Biosciences 62: 237-251.

Nour, El-Sayed. 1984. "Parity-specific Fertility Tables." Proceeding of the American Statistical Association, Social Statistics section, PP.187-192.