MODELS FOR INFANT MORTALITY IN DEVELOPED AND DEVELOPING COUNTRIES WITH APPLICATION TO A. R. E.

By

MEDANY D. MOSTAFA, AHMED H. EL MAWAZINY,

HUSSEIN ABDEL AZIZ HELMI

INTRODUCTION

Studies of deaths that occur during the first year of life, and which represent a high proportion of the total deaths, especially, in developing Countries such as the A.R. E., are of considerable interest and vital importance in our world nowadays. This is due to the fact that the success in decreasing this rate of deaths will result in great implications on both death rates, and population growth rates in general.

Furthermore, it provides a clear evidence of the society development in the economic, social, and health welfare fields, and to what extent the efforts of the public authorities are effective in this field.

This paper aims to conduct a comparative study of the types of infant mortality in the A. R. E., and some developed countries such as the U. S. A. and England & Wales. The methods adopted by this study to attain its goal will be :

1. Specification of formula that simulate the types of infant mortality of the above mentioned countries, which take the form of decreasing exponential Curve.

2. Reasonable assessment of these formula's parameters from the statistical point of view by applying the least square method on the data of the average batches of the years (54, 55, 1956), (59, 60, 1961) and (64, 65, 1966).

The paper begins with brief historic review of infant mortality rates and the accuracy of the data used in formulating these rates, in the A. R. E., as well as the developed Countries. Hence, we deal the construction of mathematical models to estimate the parameters involved in the simulation of these rates. A study of the differences between the alternative models is conducted espacially with regard to the causes of deaths, and its age-distribution during the first year of life.

The infant mortality rates differ in the various parts of the world according to the economic and welfare level of these parts, We find that while it reached in some countries, such as Sweden, about 15.4%. in the period 60-1964, it reached high levels in other parts in Equatorial African Countries where this rate was 200%. In the A. R. E., 117.4%, in India (registration area) 80.5% and in Mexico 69.3%.

The study of the data comprising numbers and rates of infant mortality reflects some indications which may be shown as follows :

1. Their general tendencies-in existence-towards the decrease in the infant mortality ratesthough this decrease fluctuates from Country to another according to the level of economic growth.

2. There are also general tendencies to reduce the difference between the rates of the infuant mortality in developed and devoloping countries.

The study of infant mortality in the A. R. E., and the changes of its rates during the last five decades shows that the A. R. E., occupies a middle situation between the various countries of the world in this respect. The infant mortality rate in the A. R. E., reached in 1966 about 127%. This rate records a decrease of about 53% when compared with the rate in 1917. This percentage of reduction is however less than the reduction observed in the developed Countries where this percentage was about 68% in the U. S. A., and 73% in England & Wales in the Period from 1920-1924 to 1960-1964. A close study of these reductions in chronological order shows a gradual and slow ratio of decrease from the rate in 1946, with the exception of 1962 when the wide-spread of Measles disease in an epidimic form caused a large number of deaths.

The study of the mortality data raises many interesting questions regarding its accuracy. In urban areas of the A. R. E., and in areas where health offices exist, the infant mortality rate is higher and reached 147%. in 1966 ; while in rural areas in general it reaches only 127% ; in the same year. These suspicions about the accuracy of the data are almost common in developing Countries as regarded by the various studies conducted all over the world.

Nature of Infant Mortality Data :

There is a kind of correlation between the accuracy of infant mortality rates and the statistical records of births and deaths during the first year of life. In this respect, the registration is considered as a main source of incorrectness of the data. Furthermore, it causes many questions and problems in the comparisons between the various areas and countries of the world. These problems are also due to the differences in definitions which refer to some vital events (such as abortion, live birth, ... etc) and differences in the methods of measuring infant mortality rates (on the basis of year of registration, or year of occurance).

The problem of registration has two main integrated aspects, namely, incomplete coverage and underregistration of births and deaths. The economic and welfare development has contributed to a great extent in dealing with the first aspect. A percentage amounts to 86% of the whole world population have been subject to registration system in 1958, and there is a constant development of this percentage. However, this welfare and cultural development, neither succeed in reducing the carelessness of registring vital events, nor in introducing accurate data.

The importance of the availability of correct and accurate data is quite clear in the previous discussion and in particular for a successful study of types of deaths. On the light of the background of statistical sources it is essential to acquaint ourselves with the degree of accuracy of death statistics in developed countries as well as in developing countries like the A. R. E., This can be achieved by using direct techniques which depend on the availablity of more than one source for the registration of vital events, hence comparison between the sources can be conducted. In the meantime, indirect techniques, which depend on the study of reasonableness of the data, its consistency and conformity by chasing the known traditional relations as characteristics of infant mortality, may be used to reach the same goal.

If we tried to study the most important indirect techniques in an attempt to assess the accuracy of data in developing countries, such as the A. R. E., and developed countries as well, we shall reach the following conclusions :

1. The time consistency of the data of developed countries is quite apparent more than that of developing countries where the infant mortality rates constanthy fluctuate in spite of the nonexistence of any abnormal conditions or enviroments that may have any effect on these rates.

2. The consistency of infant mortabity rates in developed countries with its general death rates is quite obvious. The data of some developing countries, however, reflects a less general death rate than that of the developed countries, while its infant mortality rates are higher than that of the developed countries.

3. The age-distribution of infant deaths takes the shape of negative exponential (half U) as shown in figure (1), for these deaths are very high during the first days of the year, then decreases gradually up to the end of the year. The sudy of infant mortality curves in the U. S. A., and England and wales ensures this fact. The percentage of infant deaths of one day old in both countries in 1966 was 42% and 35% respectively, while this percentage in Mexico, for instance, reached about 5% only in the same year. In the A. R. E. it was about 0.3%, 0.2% 1.8% for 1960 male infant deaths in rural areas, with medical centers, rural areas without medical centres, and urban areas respectively. The same conclusions may be reached by studying infant deaths of few weeks old infants in the developed countries and the A. R. E.

This may lead us to think that this type could be due to either the nature of the data, or the basic difference between age-distribution of infant deaths of both sexes in developing and developed countries.

4. Study of sex-ratio evolution on the basis of the fact that, it is well known that male deaths are usually higher than female deaths at all ages. Its percentage is about 100-110% and if it reaches 130-140% it is usually accompanied by low death rates. The study of the data of the U. S. A., and England & wales ensures this fact, where the percentage ranges from 135-138%. It is noticed that the study revealed that this percentage reaches its utmost peak during the beginning of the first year of life. This percentage in the A. R. E. in 1966 was

about 98%, thus, it is less than the reasonable percentage and raises many suspicions about the nature of the data, These suspicions turn our attention to study this percentage in a more developed area such as Cairo Governorate, in which this percentage reaches 101%, and characterized with stability during the batches of years to be studied. The detailed study of this ratio indicates the following points :

(*a*) Sex-ratio of infant death rates of 3 weeks old which is about 129-137% may be similar and comparable to the ratio in the developed countries.

(*b*) Sex-ratio begins to decrease less than 100% from the second month up to the end of the first year. This conclustion causes some suspicions about the accuracy of Cairo Governorate data.

Evaluation of infant deaths data in the A. R. E. :

The study of the deaths data reveals its incompleteness particulary, when we compare the data of urban and rural areas with medical centres with the similar data of the rural areas without medical centres. This comparison indicates that the infant mortality rates in the first are more than its value in the second. This is due to under-registration. The evaluation studies of births and deaths has proved that this under-registration in rural Lower Egypt reaches about 127.8% during the first thirteen days of life while it reached 118.6% in rural upper Egypt according to the data of the average years 1959, 60, 61. The estimated percentage of births under registration in some of governorates in Lower and upper Egypt reached about 8.5% and 5.8% respectively.

For all these reasons, research workers in vital. Statistice studies should not rely on the A. R. E. date in repesenting the actual type of infant mortality. This was the main reason which made us confine ourselves to the Cairo Governorate date to represent the actual type of infant mortabity in the A. R. E. This decision was based on the results of the study of the sex-ratio in the governorate during the first thirteen days and was shown to approach the universal level.

Furthermore, the Cairo Governorate data is more consistent and accurate than any other part in the A. R. E. in the meantinme ; it is similar to the other parts of the A. R. E. in the type of infant mortality This pattern of infant mortaliy rates are usually high at the beginning of life, then it decreases gradually with the elapse of the first month, then it begins to increase from the 2nd month up to 4th or 5th month, after that it decreases again up to the end of the first year.

TABLE (1)

The Proportion of Consecutive Intervals for

Infantile Mortality Rates in U.S.A.

(1963)

Death age Interval		Frequency (Infantile-mortality rates)	Adjusted Frequency	The Proportion of consecutive int.
0— 2		1365	13645	12,05
3— 6		151	1132	3,55
7—13	days	74	319	1,91
14—28		78	167	1,37
29—59		130	122	1,20
2—		102	102	1,34
3—		76	76	1,33
4—		57	57	1,36
5—		42	42	1,24
6—	months	34	34	1,17
7—		29	29	1,26
8—		23	23	1,15
9—		20	20	1,05
10—		19	19	1,12
11—12		17	17	

Source : U. S. Department of Health, Education and Welfare.

Public Health Service.

TABLE (2)

The Proportion Between Consecutive Intervals
between Infant Mortality Rates in England-Wales
(Years average 1958, 1960, 1962)

Deaths Age Intervals		Frequency (Infantile mortality Rates)	Adjusted Frequency	Proportion between Consecutive Intervals
0— 2		1127	11270	7,19
3— 6		209	1568	3,27
7—13	day	112	480	2,03
14—28		110	236	1,82
29—59		139	130	1,12
2—		116	116	1,22
3—		95	95	1,37
4—		70	70	1,35
5—		52	52	1,27
6—		41	41	1,17
7—	month	35	35	1,75
8—		28	k8	1,12
9—		25	25	1.14
10—		22	22	1.16
11—12		19	19	

Source : Population Studies. Vol. I, 96.

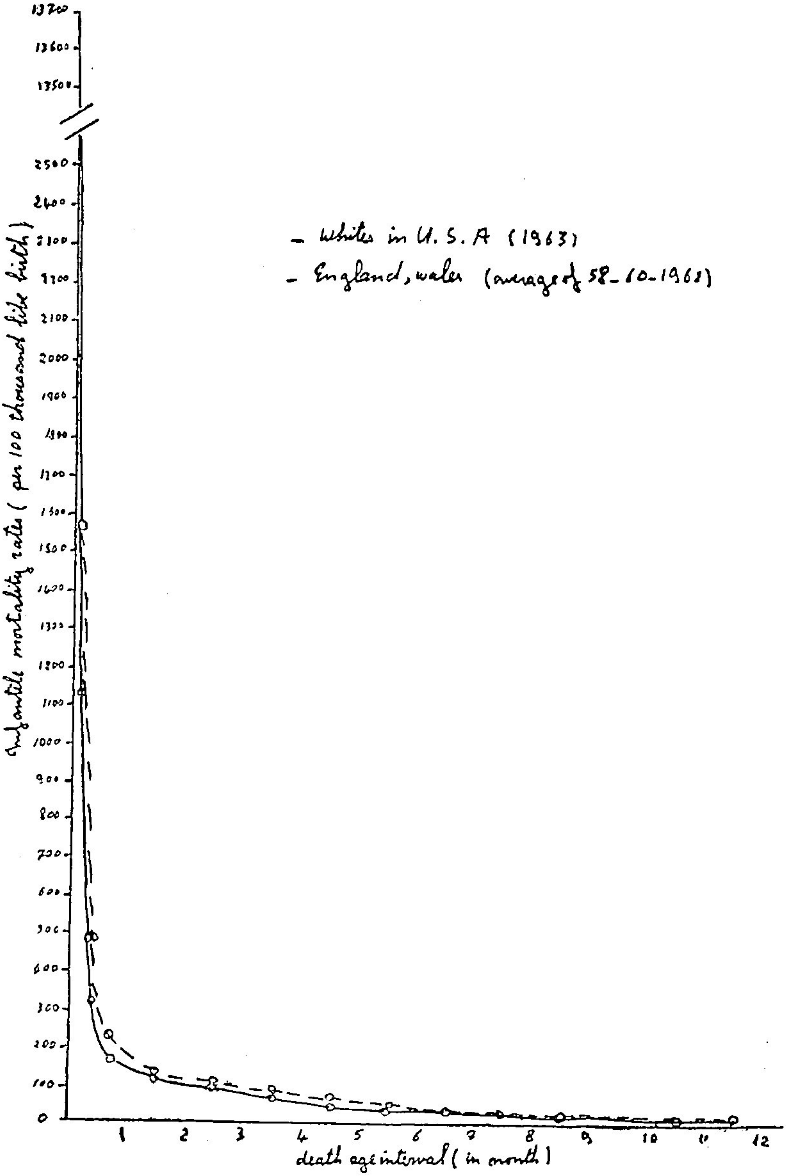
- Whites in U.S.A (1963)
- England, wales (average of 58-60-1961)
Infantile mortality rates (per 100 thousand life birth)
death age interval (in month)

figure (1)

Infant Mortality Type in Developed Countries :

Infant mortality rates curves in developed countries such as U. S. A. and England & Wales attain its highest value in the first days of the infant life, then it decreases quickly during the first two weeks, after that, these rates are subject to gradual decrease up to the end of the year. Of course, its percentage differs from one country to another.

The study of ratios of the mortality rates at the centers of succssive classes ensures the previously described type. It also indicates that the infant mortality rates in the U. S. A. tends to relative stability from the middle of the first month of infant's life. This relative stability is achieved in England & Wales after the end of the first month.

In an attempt to fit a curve which represents the infant mortality rates, we find that the irregularity at the beginning of the data (which is due to the relative quick decline of the mortality rates) will have great effects on the estimated type. This is due to the high percentage of deaths which occur during the first two weeks and was about 71.2% and 65.8% in the U. S. A. and England & wales, respectively. This indicates the difficulty of is fitting especially because the number of classes within this period are very little (3 classes only). For that reason, we decided to start the curve fittings for the available data from the beginning of the stability period, which is the middle of the first month of the infant life.

By recognizing the shape of the curve during its regularity, we find that it takes the form of decreasing exponential curve type. It means that it follows the following formula :

$$Y = A\ e^{-\alpha x} \quad (1), \text{ where :}$$

X is the age at death by month,

Y is the Number of deathes at age X,

A, a are positive parameters.

The parameters of his formula may be estimated by using the method of least square.

TABLE (3)

The Proportion of Consecutive Intervals for Infant Male deaths in A.R.E.

Deaths Age Interval	Years average (54, 55, 1956)		Years average (59, 60, 1961)		Years average (64, 65, 1966)	
	adjusted frequency	the proportion of consecutive intervals	adjusted frequency	the proportion of consecutive intervals	adjusted frequency	the proportion of consecutive intervals
0— 2	9870	1,66	10310	1,73	9230	1,57
3— 6	5963	1,90	5970	1,73	6200	1,79
7—13 day	3146	2,33	3446	2,27	3510	2,19
14—30	1474	1,37	1515	1,53	1605	1,69
1—	1047	,86	988	,92	947	,89
2—	1245	,80	1075	,82	1069	.82
3—	1565	,98	1304	,95	1299	,93
4—	1603	1,08	1375	1,11	1392	1,10
5—	1481	1,03	1234	1,01	1269	,97
6— month	1442	1,19	1219	1,01	1303	1,17
7—	1212	1,08	1104	,96	8116	,99
8—	1120	1,20	1060	1,12	1028	1,11
9—	935	1,15	950	1,19	929	1,26
10—	814	1,30	800	1,18	738	1,10
11—12	624		678		670	

TABLE (4)

The Proportion between Consecutive intervals for

Female Infantile deaths in A.R.E.

Deaths Age intervals	Years average (54, 55, 1956) adjusted frequency	the proportion between consecutive intervals	Years average (59, 60, 1961) adjusted frequency	the proportion between consecutive intervals	Years average (64, 65, 1966) adjusted frequency	the proportion between consecutive intervals
0— 2	7180	1,56	7600	1,57	7130	1,44
3— 6	4598	1,81	4815	1,82	4943	1,56
7—13 day	2537	1,76	2640	1,91	3167	1,97
14—30	1438	1,29	1380	1,27	1609	1,14
1—	1115	,81	1089	,89	979	,83
2—	1384	,83	1223	,85	1186	,81
3—	1658	,97	1438	,98	1465	,94
4—	1703	1,05	1466	1,15	1552	1,11
5—	1618	1,03	1289	,98	1404	1,03
6— month	1565	1,18	1312	1,13	1358	1,09
7—	1326	1,09	1157	,99	1243	1,11
8—	1215	1,16	1167	1,14	1124	1,07
9—	1051	1,11	1026	1,15	1046	1,23
10—	947	1,34	893	1,17	851	1,10
11—12	705		772		773	

TABLE (5)

The Proportion between Consecutive Intervals for Infantile Mortality Rates in A.R.E.

Deaths Age Intervals	Years average (54, 55, 1956) adjusted frequency	Years average (54, 55, 1956) proportion between consecutive intervals	Years average (59, 60, 1961) adjusted frequency	Years average (59, 60, 1961) proportion between consecutive intervals	Years average (64, 65, 1966) adjusted frequency	Years average (64, 65, 1966) proportion between consecutive intervsa
0— 2	8450	1,61	8980	1,66	8210	1,41
3— 6	5295	1,86	5400	1,77	5640	1,69
3—13 day	2850	1,96	3051	2,11	3343	2,08
14—30	1457	1,33	1449	1,40	1607	1,66
1—	1094	,83	1038	,90	963	,86
2—	1312	,81	1148	,84	1126	,82
3—	1611	,98	1370	,96	1380	,94
4—	1652	1,07	1420	1,13	1470	1,10
5—	1548	1,03	1257	,99	1335	1,60
6— month	1502	1,18	1265	1,12	1330	1,13
7—	1268	1,09	1130	1,02	1178	1,10
8—	1166	1,18	1113	1,13	1075	1,09
9—	992	1,13	987	1,17	986	1,24
10—	879	1,33	846	1,17	793	1,10
11—12	663		724		720	

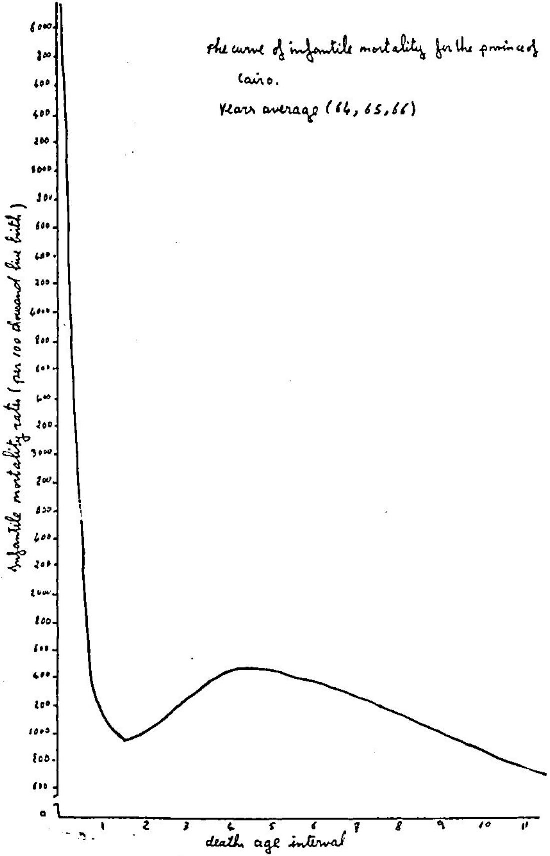
the curve of infantile mortality for the province of Cairo.
Years average (64, 65, 66)
Infantile mortality rate (per 100 thousand live birth)
1 2 3 4 5 6 7 8 9 10 11
death age interval

figure (2)

(A) *Parameters estimated by using least square method :*

If we take the Logarithm of both sides of the previous formula we shall have the following equation :

$$F_r = B - a\, x_r \qquad (2), \text{ where :}$$

F_r is the logarithms of Y and

B is the logarithms of A.

If we assume that there is a random error 6, then the variables follow the following formula :

$$F_r = B - \alpha X_r + \epsilon_r \qquad (3), \text{and}$$

$$\sum_{r=1}^{n} \epsilon_r^2 = \sum_{r=1}^{n} (F_r - B + \alpha X_r)^2 \qquad (4)$$

Differentiating the above expression for the error sum of squares with respect to the parameters B &α and equating to zero to obtain the estimators based on minimum error sum of squares, namely ;

$$\hat{\alpha} = \frac{\sum_{r=1}^{n} F_r X_r - \frac{1}{n}\left(\sum_{r=1}^{n} F_r\right)\left(\sum_{r=1}^{n} X_r\right)}{\frac{1}{n}\left(\sum_{r=1}^{n} X_r\right)^2 - \sum_{r=1}^{n} X_r^2} \qquad (5)$$

$$\hat{B} = \frac{1}{n}\sum_{r=1}^{n} F_r + \frac{1}{n}\hat{\alpha} \cdot \sum_{r=1}^{n} X_r \qquad (6)$$

The estimator of the parameter A may be obtained from the relation $\hat{B} = \ln \hat{A}$

1.—*Infant Mortality Formula for the U. S. A. :*

Adopting the previous technique, we may obtain the least square estimators of the parameters of the suggested formula for the infant mortality in the U. S. A. The data of 1963 led to the following formula :

$$Y = 166. \quad e^{-0.216X} \qquad (7)$$

From this formula we may obtain the expected infant mortality rates in the age between (X', X'') by integrating to obtain the area under the curve bound between the X — axis and the ordinates X^1

and X'', i.e.

$$\hat{X}_n = \int_{X'}^{X''} y \cdot dx = 166 \int_{X'}^{X''} e^{-0.216X} \cdot dx$$

The comparison of the expected rates obained from the above formula, with the original data and the study of the percentages of errors in estimation, indicated the existence of large difference in some classes. On the other hand the Chi-Square test proved the efficiency of this formula in representing infant mortality type in the U. S. A. in 1963. The following numerical results was obtained :

Chi-Square Calculated from the data = 6.97

Chi-Squara from the tables with 9 degrees of freedom and 5% error = 16.916.

This indicates the insignificance of the null hypothesis, and shows the conformity between the original distribution and estimated rates of infant mortality during the first year of life.

2.—*Male and Female Infant in the U. S. A. :*

The study of male & Female infant mortality types shows the conformity between the type which the formula number (1) represents for both males and females, of white and coloured infants, from the beginning of the middle of the first month up to the end of the year. Different Parameters was obtained for each group.

Studying the results of the expected rates obtained from the formulas, representing the type in each case, showed the insignificance of null. hypothesis by the Chi-Square test. Therefore we may conclude that the original distributions conform with estimated distribution.

In addition the study reveals the following points :

(a) The existence of a difference between the rates of white and coloured, this may be due to the difference in the levels and standards of health welfare.

(*b*) There are some suspicions about the accuracy of the coloured infant mortality data, when we take into consideration the sudden increase of the deaths in the 12th month of the infant age.

In spite of the fact that these curves were based on the 1963 data only. We may assume that the formula obtained represents the infant mortality during the years arround 1963, if we takes into account the relative tability of its death rates.

Infants Mortality Formula in England & Wales :

The similarity between the type of infant mortality rate curve in England & wales, and the type of infant mortality in the other developed countries leads us to use equation (1) for fitting its infant mortality data. The least square method was used to estimate the paremeters of the equation and the following formula was obtained :

$$Y = 200. \quad e^{-0\ 221\ X} \qquad (8)$$

The results of the Chi-Square test ensures the insignificance of the null-Hypothesis, and accordingly the conformity between the estimated distribution of infant mortality rates and the original for the average of years 1958, 60, 62. It may be noticed that the degree of conformity is less in this case than in the case of the U. S. A. data. This is because the Chi-Square in the case of England & Wales was calculated to be 15, 60, while it was 6.97 for the U. S. A. data.

On a trial to find an improvement to the fitting we find that the irregulaity with its relative quick decline, affects the efficiency of the estimated curve in England & Wales. This may lead to think that it is better to confine the representation of equation (1) to the post-neo natal mortality only-In this case, equation (1) proved to be an excellent fit for the data, this was reflected by a calculated Chi-Square equals to 3.29 only.

Infant Mortality Type In the A. R. E. :

The study of the date of infant mortality rates in the A. R. E. shows that these rates reach its maximum values during the first days of life. I begins to decline afterwards during the first month, then it begins to increase from the 2nd month up till the fourth or fifth month, then it decreases gradually up till the end of the first year. The study

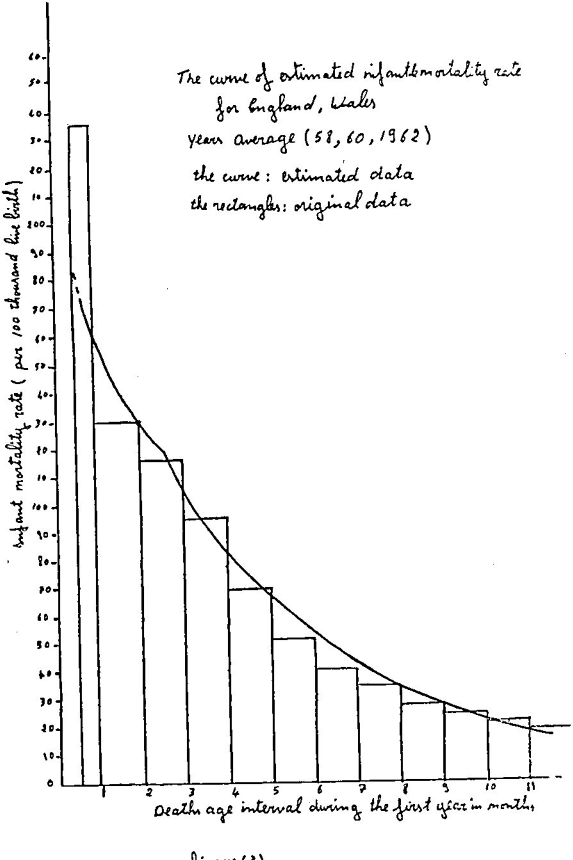
The curve of estimated infant mortality rate
for England, Wales
years average (58, 60, 1962)
the curve: estimated data
the rectangles: original data
Infant mortality rate (per 100 thousand live birth)
Death age interval during the first year in months
0
1
2
3
4
5
6
7
8
9
10
11

figure (3)

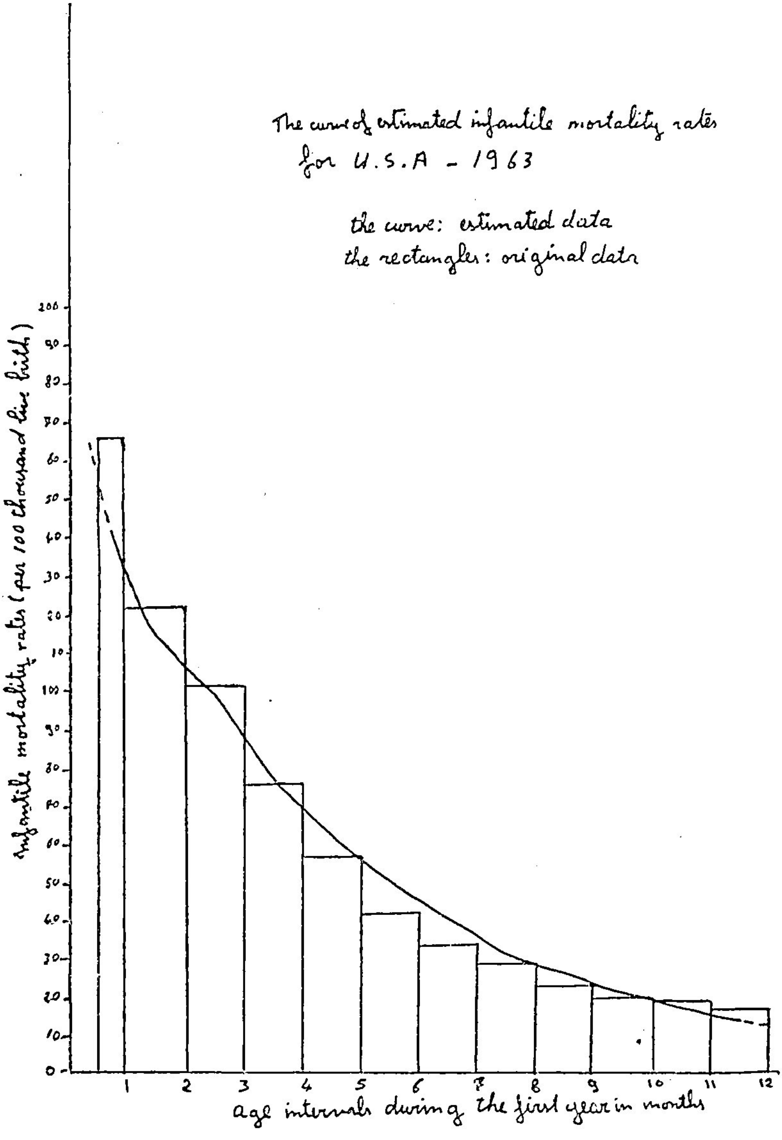
The curve of estimated infantile mortality rates
for U.S.A - 1963
the curve: estimated data
the rectangles: original data
infantile mortality rates (per 100 thousand live birth)
age intervals during the first year in months
1
2
3
4
5
6
7
8
9
10
11
12

figure (4)

of the ratios of the successive classes of infant mortality rates during the first year ersures his pattern, not only that, but it indicates its conformity with regard to the batches of the years under study, which are (1954, 55, 56), (1959, 60, 61), (1964, 65, 66).

From the above discussion and recalling the patterns of infant mortality rates in developed countries which is always decreasing, it is obvious that the model suggested earlier for developed countries will not work for the A. R. E. or countries with the same pattern. The pattern of infant morality rates in the A. R. E. leads us to think of the following model :

$$Y = Ae^{-\alpha X} + B X^{C} e^{-\beta X} \qquad (9)$$

where : X is the age in monthes,

Y is the number of deaths at the age X, and

A, B, C, B, α are the parameters of the equation.

We shall try to estimate these parameters by using the least square method on the basis of the following assumptions concerning the two components of the previous formula :

(*a*) The eflect of the first component of the formula is very large at the beginning of the curve, then its value decreases at high rates.

(*b*) The effect of the second component of the formula is very small at the beginning of the year, but it increases gradually to reach its maximum, and then begins to decline once again gradually until it becomes the main factor in infant mortality rates after the 2nd month.

We must take into consideration in any attempt to estimate the parameters of the previous formula, that its first part resembles the formula representing infant mortality type in developed countries. For that reason we shall begin with the data of the first few classes to obtain the first estimators of the parameters of first part of the model, namely ; α and A. The expeced number of deaths due to this part of the equation may be obtained by integration over the appropriate period of age. Therefore, the following relation follows which give the expected number of deaths due to the second component :

$$\left(y_n - \int \hat{A}\, e^{-\hat{\alpha} . x}\, dx \right) = B\, X_n^{C}\, e^{-\beta X_n} \qquad (10)$$

In general, the sth approximation to the estimators of the parameter A and α ; namely ; A_s and α_s based of the $(s+1)$ *st* approximation of the estimators of the parameters β , B, C, are obtained from the equation :

$$\left(y_n - \int \hat{B}_{(s-1)}\, X^{C(s-1)}\, e^{-\beta_{(s-1)} X}\, dx \right) = A\, e^{-\alpha x_n} \qquad (11)$$

While the sth approximations of β , B, C ; namely ; $\hat{\beta}_s$ $\hat{B}_s$ $\check{C}_s$ are based on the equation :

$$\left(y_n - \int \hat{A}_s\, e^{-\hat{\alpha}_s x}\, dx \right) = B\, X_n^{C}\, e^{-\beta X_n} \qquad (12)$$

The estimators will be based on the minimum error sum of squares and obtained from the previous two equations after taking the logarithm of both sides in each case as follows :

a) *The estimates of the First part Parameters :*

We may notice the resemblance between the first part of equation (9) and the model suggested for the developed countries as given by equation (1). The parameters A and α will be estimaed on the basis of least square similar to the procedure given earlier and led to equations (5) and (6)..

(b) *The estimates of the second part parameters :*

From the formula of the curve which represents the first part, we have the values of Y which represents the different values of X, which in its turn, interrelates with the second part in a part of the curve. Therefore, it must be excluded to estimate the parameters of the second part. For this we may assume that :

$$Z = Y - Y^* = B\,X^C\,e - \beta^X, \tag{13}$$

where :

Y is the actual observations,

Y* the estimated values originated from the formula of the first part.

If we take the logarithm of both sides of equation (13), we shall have the following model :

$F = D + C \log - \beta X$, where :

$F = \log Z$, and

D is the logarithm of B $D = \log B$

If we assume that here is a random error, E , the variables will follow the following formula :

$$F_r = D + c \log X_r - \beta X + \varepsilon_r$$

Thus the error sum of squares follows from the equation :

$$\sum_{r=1}^{n} \epsilon_r^2 = \sum_{r=1}^{n} \left(F_r - D - c \log X_r + \beta X_r\right)^2 \tag{14}$$

Differntiating the above formula partialy with respect to the parameters, which are D, C, β and equating to zero for minimum error . Sum of squares, we have the following formulas :

$$\sum F_r = n\hat{D} + \hat{c} \sum \log X_r - \hat{\beta} \sum X_r \tag{15}$$

$$\sum F_r \cdot \log X_r = \hat{D} \sum \log X_r + \hat{c} (\sum \log X_r)^2 - \hat{\beta} \sum X_r \cdot \log X_r \tag{16}$$

$$\sum F_r \cdot X_r = \hat{D} \sum X_r + \hat{c} \sum X_r \log X_r - \hat{\beta} \sum X_r^2 \tag{17}$$

These formulas may be solved to obtain the estimators of parameters.

A study of the percentages of successive classes ratios indicates that :

(*a*) The percentages of the first three classes (0-2, 3-6, 7-13 days) form a certain stage ; in this stage the ratio increases, that means the death rates decrease. Then, the ratio begins to decrease suddenly in a way that provides the possibility of using these classes to estimate the values of the parameters of the first part of the curve.

(*b*) The trends of the 4th class (14-30 days) approach the previous classes trends in a way that it can be considered within the first part of the curve, though it is our intention to exclude it when estimating the value of the parameters of this part, to secure the consistency of data and to eliminate the effect of the interrelated second part of the curve.

(*c*) The successive classes ratio from the 2nd month up to the end of the first year, form another stage, where the rates increases gradually untill the 5th month, after that it decreases to the end of the year. Thus it provides the possibility of using it in estimating the parameters of the second part of the curve. It is noticed that we shall exclude the interrelations (frecquencies) resulted from the first period when we take into account that these data will be actually representing the infant mortality of the second part only.

We shall use this method in estimating the parameters of the formula which was suggested to represent the mortality type in the batches of years.

A) *A Formula of Total Infant Mortality in the A. R. E. (for the average of the years 1959, 60, 61) :*

By applying the previous method on the Cairo Governorate data, we obtained the estimates of the parameters of the two parts of formula (9) which compose the infant mortality curve, by using the least square method.

Thus the following formula was obtained :

$$Y = 10400^{-3.56X} + 884\ X^{0,9226}\ e^{-0.2076X} \qquad (18)$$

The infant mortality rates of this formula has been estimated by using the computer 1620 by a programme which appears in the appendix, this programme is prepared to calculate the area under the curve. The results of this process indicate :

1. The consistency of the results with our assumptions concerning the contributions of the parts composing the mortality cruve from the point of the intensity of the first part effect at the beginning of the year and its decline from the beginning of the 2nd month, while it is quite clear that infant mortality rates estimated by the second part, represent the basic part of mortality formula since the beginning of the 2nd month up to the end of the year.

2. It was noticed, by studying the error percentages of estimation, that these percentages are little more than 10% in one class between 5% in three classes, and less than that in the rest of classes. Thus formula (18) is considered to be a good model (from the statistical point of view) for the infant mortality type for the average of years under study. In this case the resulls of the Chi-Square test does not agree with the above conclusion and reflect the significance of null hypothesis of the test. This leads to the inconformity of the original and estimated distributions. This is basically due to the effect of the 3rd class (7-13 days) which was about 27% of the calculated value of Chi-Square, this later value was 34.92, which is larger than the tabulated chi-square of 27, 877, with error 1% and 9 degrees of freedom.

3. With view to the nature of infant mortality data in the A.R.E. the great suspicions about the accuracy of the correctness of the age registeration at death of the infant according to the given classes, and the possibility of its being in the previous or later classes, we may combine Some classes during the frist month, and limit it to two classes only (0-6, 7-30 days), which will result in changes of the chi-square test results in this case, and indicate the conformity of original and estimated distributions because :

Chi-Square caleulated value is 23, 71, while

Chi-Square from tables on the basis of 7 degrees of freedom and error 1%. equals 24, 322.

TABLE (6)

Estimating Parameters of Equations Describing the Infant Mortality during the Years Studied.

	First Approximation					Second Approximation				
	$\hat{a}$	$\hat{a}$	$\hat{b}$	$\hat{c}$	$\hat{\beta}$	$\hat{a}$	$\hat{\alpha}$	$\hat{b}$	$\hat{c}$	$\hat{\beta}$
Years average (54, 55, 1956)										
Male	11700	3,70	8 8	1,3204	,30 4	11900	4,02	911	1,2245	,2887
Female	8330	3,45	939	1,2371	,2821	8570	3,74	957	1,2099	,2788
Total	10100	3, 5	91	1,2432	,2867	10300	3,93	934	1,2485	,283
Years average										
Male	11900	3,60	812	,9665	,2201	12000	3,88	828	,9588	,2174
Female	8940	3,51	963	,8333	,1935	9170	4,02	989	,7922	,1844
Total	10400	3,56	884	,9226	,2075	10600	3,92	910	,8745	,2025
Years average										
Male	10900	3,22	711	1,1922	,2604	11000	3,41	749	1,1518	,2543
Female	8020	2,69	725	1,3016	,2712	8100	2,78	1,2372	1,2372	,2618
Total	9480	3,00	729	1,2260	,2627	9580	3,14	748	1,2050	,2578

TABLE (7)

The Results of X Test in Different Cases

	First approximation			Second approximation		
	X along the curve	X on the basis of confounding	X for monthly intervals	X along the curve	X on the basis of confounding	X for monthly intervals
Years average (54, 55, 1956)						
Male	44,46	22,50	22,00	46,89	24,46	21,25
Female	45,28	21,83	21,68	46,80	20,01	15,34
Total	36,53	15,44	15,29	60,94	38,50	34,70
Years average (59, 60, 1961)						
Male	42,17	27,84	27,63	36,23	20,91	18,36
Female	40,24	31,70	28,80	32,16	21,62	20,70
Total	34,92	23,71	22,63	31,17	19,28	17,99
Years average (64, 65, 1966)						
Male	28,92	19,53	19,61	20,80	17,99	17,47
Female	19,93	15,46	15,14	18,82	13,08	12,92
Total	22,96	22,84	17,81	15,51	14,69	13,02

X' from the table with 9 degrees of freedom and level of significance = 001 % = 27,877.
X' « « « « 7 « « « « « « = 001 % = 24,322.
X' « « « « 6 « « « « « « = C01 % = 22,457.

B) *The successive iteration :*

We shall use this method to obtain more accurate estimates of the parameters. This will be achieved by the exclusion of the interrclation between frequencies of each of the first and second parts of the infant mortality formula curve. The basis of the estimation of the parameters of each part which appears in table (6) will be to use the frequencies due to this part after eliminating the frequencies due to the other part as estimated by the method shown earlier. We may obtain the estimates of the 2nd appcoximation of the parameters of equation (9) by using equation (11) and (12). Successive iterations leads to the formula which represents the mortality pattern based on the average of the data in 1959, 60, 61. This formula is :

$$Y = 10 \quad e^{-3.92X} + 910 \; X^{0.8745} \quad e^{-.2025X}$$

This formula differs from that given by equation (18), particularly ; with regard to the estimators of the parameters, of the first part. Adding some classes of the first month, as previously shown, we shall find out that the result of the Chi-Square test reflects the conformity of original and estimated distributions in this case also.

In the light of these results, we can say that equation (9) satisfactory represents from the statistical point of view, infant mortality type in the A. R. E. (on the basis of the Cairo Governorate data) for the average of the years (1959, 60, 61), (1964, 65, 66) and (1954, 55, 56). These latter years formulas may be obtained by he same method.

C) *Formulas for male-female infant mortality :*

We may estimate the parameters of the formulas representing male and female infant mortality for the batches of the years under study by iterations using the least square method and on the basis of the same assumptions previously stated. From these formulas we may estimate the infant mortality rates during the first, year, and compare it with the original data.

The studies of error percentage in the estimated values and the Chi-Square tests have shown the conformity of the original distribution with the estimated distribution. Therefore formula (14) represents a satisfactory infant mortality type for the batches of the years under study on the basis of the Cairo Governorate data.

D) *The Final Results :*

The least square method and successive iterations method were used to obtain estimates of he parameters of total, male and female infant mortality formulas. The results of the study of the error percentage in estimation, and the results of Chi-Square tests was considered in three different cases according to the grouping of the data. The first case was the grouping of the first month data as follows :

(0-2), (3-6), (7-13), (14-30) days. The second case was obtained by combining the data of the first month into two groups : (0-6) and (7-30) days. The third case, the data of the first month was taken as one group. In all the above cases the rest of the year was taken on monthly basis.

The above two test criteria showed that :

(*a*) The first approximation was quite adequate in representing total and female infant mortality rates, while the 2nd approximation was suitable in represerting male infant mortality type for the batch of years (1954, 55, 56).

(*b*) The second approximation is more efficient in representing infant mortality type for the two batches of years (1959, 60, 61) and (1964, 65, 66).

A comparative Study of Infant Mortality Types in Developed Countries and the A. R. E.

A.—*Study of differences :*

The study of frequencies and rates of infant mortality in devetoped countries and the A. R. E. indicates that there are differences between the patterns of infant mortality. These differences are due to the increase of infant mortality rates in the A. R. E. during the period from 2nd — 6th month, and its gradual decline after that, whereas the infant mortality rates in developed countries tend to decrease constantly since the beginning of the year untill its end in the form of negative exponential curves (half U).

It is necessary to study age-distribution of these deaths during the first year, and its relation to the death causes in an attempt to explain this diflerence between the types of infant mortality. In this

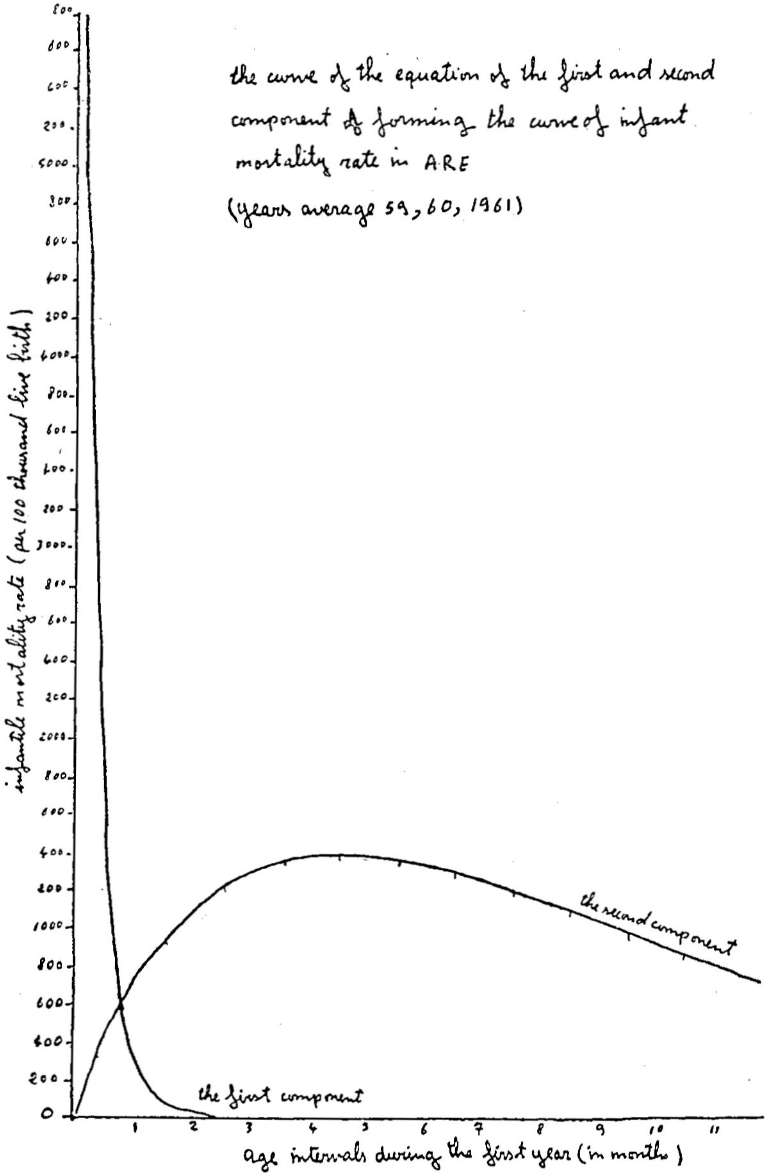
the curve of the equation of the first and second
component of forming the curve of infant
mortality rate in ARE
(years average 59, 60, 1961)
infantile mortality rate (per 100 thousand live birth)
the second component
the first component
age intervals during the first year (in months)

figure (5)

connection it is possible to decompose infant mortality into two components as shown in Figure (5), these components may be explained as follows :

1. The first component is the exogenuous infant mortality which comprises those cases in which the infant picks up the factor which causes its death from the environment in which it lives and it may be regarded as accidental in the broader sense of the term. Its effect reaches its highest intensity during post-Neo-Natal Mortality, that is, during the period from the 2nd month up the end of the year, Furthermore its effect may be extended to the first month of the infan age.

2. The second component is the end-ogenuous infant mortality which comprises those cases in which the child bears with itself from birth, the cause resulting in its death, whether that cause was inherited from its parents at conception or acquired from its mother during gestation or delivery. Its highest effect takes place during the first four weeks, or the first month in general, when the effect of enviromental factors in which the infant lives weakened, and the effect of the other factors, which the child bears from birth increases.

By considering these two components which form the partial distribution of infant mortality rates over the periods of the 1st year of life, we find out that while the 1st component is characterized with a high degree of flexibility, and there is an increasing possibility of excercising control on it by affecting the enviromental factors, the possiblity of affecting the second component is very little, this is due to inheritance considerations.

Though this classification of infant mortality depends basically
Though this classification of infant mortality depands basically
on the statistics of death causes, we shall use Bogeois-Pichat Measure, which is called. The Biometric Measure» in making this classification. This measure depends on the previous assumption about the time range of the effect of each component, on the basis that we shall consider all deaths occuring during the last 11 months of the first year, are due to external causes. Accordingly the age distribution of deaths is considered unchangeable with the change of time. It is considered seperate from the infant mortality level, that means in other words, that the death ratio in any of these months will be unchangeable. In

this case, we can have the exogenuous infant mortality ratio during the first month, estimated by Pichat, while reaches about 25% of the total deaths of the last 11 months, by using mathematical extrapolation or diagram extrapolation.

By studying the extent of conformity between the main assumption of the measure, that is, the unchanging of death ration in any month in relation to the total deaths of the last 11 months, and the A. R. E. data, we find out that it is difficult to say that these ratios are unchangable during the batches of years under study however, it can be considered approximately close to each other. This situation is different from the situation prevailing in developed countries, where are approximately equal the ratios in the U. S. A. & England and Wales.

B) *Infant Mortality & Distribution by Pichat Measure :*

The infant mortality rates of the average of the years (1964, 65, 66) were classified according to its causes by using the previous measure.

It was noticed that the exogenuous infant mortality rates are about 15444, while endogenuous infant mortality rates are about 127 only (for every 100 thousands livebirths). These rates in developed countries are : In the U. S. A. about 686 for exogenuous infant mortality, and 1531 for endogenuous infant mortality, and in England & Wales they were 802 and 1398 respectively for the average of the years (58, 60, 62).

No doubt that this age-disribution of infant mortality shows the great difference between its patterns in developed countries and the A. R. E. while Exogenuous death ratio in the first ranges from 31—36%, we find that it reaches about 99% in the second. This reflects the great effect of the enviromental and urban conditions on death causes, in the meantime, it indicates the available potentialities in the A. R. E. to decrease the infant moratlity by controlling the exogenuous infant mortality.

It may be noticed that by applying the measure on the A. R. E. data, we have in some cases meaningless results (exogenuous infant mortality rate is higher that total infant mortality rate) This may lead us to discuss the appropriatness of this measure to the conditions and circumistances prevailing in developing countries from two sides : to what extent its basic hypothesis take into consideration these conditions, and the accuracy of its statistics.

C) *The Relation Between Pichat Classifieation and Formulas Represent ing Infant Mortality Types :*

The Formula (1) represents infant mortality type in developed countries from the middle of the first month up to the end of the first year of infant age. According to Pichat classification this formula represents exogenuous infant mortality. If we compare the results of this formula with the results obtained using the measure, we shall find little differences between them, This difference was 6.4% and 8.6% for the data of England & wales and the U. S. A. respectively. This provides the possibility of accepting this formula as a representative of this kind of deaths in developed countries. In the meantime we find that this formula does not represent the endogenuous infant mortality, which usually takes the form of unchangeable ratio of the total deaths. This is due to its non-response to health developments in these countries which reached a high level of progress.

As regards infant mortality pattern in the A. R. E. which is represented by formula (14), we can reach the following conclusions in the light of the facts that have been revealed and proved during the study of the two parts of this formula :

1. The 1st part is considered as a representative of endogenuous infant mortality. Its ratio to the total deaths reaches about 19% (according to the estimated results of the formula of the average of the years (64, 65, 66), comparable with the ratio reached by Pichat Measure, (about 0.8%). Furthermore, this study indicates that these rates are small in comparison with these in developed countries. It shows also its gradual and slow decline.

2. The 2nd part of the formula is considered as a representative of exogenuous infant mortality, its ratio to total deaths is about 89% according to the formula for the average of the years (1964, 65, 66) while the ratio reaches about 99% according to Pichat Measure.

These results shows that it is not appropriate to use Pichat measure in developing countries, particulary in the light of what the estimated formulues representing the infant mortality pattern in the A. R. E. has shown, of contributing the exogenuous infant mortality with 2.3% of the total deaths in the last 11 months for the average of the years (64, 65, 66).

D) *Conclusion :*

The above results indicates that exogenuous infant mortality represents the main difference between the types of infant mortality in developed countries and the A. R. E. with we find that the welfare development in developed countries has great implications on the rate of this part, and hence on the total death rate. We notice with regard to the A. R. E. an increase in the infant mortality rates that begins at the 2nd month to reach its maximum during the period from 3-5 months of the infant age.

If we tried to relate this increase of infant mortality rates to the stages of infant life, we shall find that, from the beginning of this period, the infant will have some external fluids, and loses gradually the natural emmunity gained directly after their birth. From now on the enviromehtal factors will affect to a great extent on infant's health and living conditions, in the meantime, the probabilities of his death increase, particulary exogenuous infant mortality. As regards the E. A. R. data, we notice that Exogenuous mortality ratio to total deaths in the period of study had decreased a little, it was 83% in the period (54, 55, 66) while it reached 81% for the average of the years (64, 65, 66). It is quite clear that this result does not reflect the great achievements of the E. A. R. in the various fields, particulary in the field of health welfare and urban development. For this reason, we have no alternative but to suspect the nature of the data which have been the basis of these ratio.

APPENDIX (1)

أولا : /

برنامج حساب التكرارات المتوقعة من الجزء الأول لمعادلة وفيات الرضع

ف ج . م . ع .

```
1. READ 2, A 1, B 1, CODE
2. FORMAT (F 5. 0, F 4. 2, 69 X, 12)
3. READ 4, A, B, C
4. FORMAT (2 F 5. 2, 68 X, F 2.0)
   IF (C—1.) 5, 1, 1
5. YB = A 1* (EXPF (B 1* B) / (—B 1))
   YA = A. 1* (EXPF (B 1* A) / (—B 1))
   RINT = YB—YA
   IF (SENSESWITCH 1) 6, 8
6. PRINT 7, CODE, A 1, B 1, B 1, A, B, RINT
7. FORMAT (11, 4 F 10.2, 4 X, F 15.3, 17 X, 2 H 99
8. PUNCH 7, CODE, A 1, B 1, A, B, RINT
   GOTO 3
   END
```

APPENDIX (2)

ثانيا :

برنامج حساب التكرارات المتوقعة من الجزء الأول لمعادلة وفيات الرضع في ج . م . ع .

```
      DIMENSIONT (10)
      COMMONB 1 C 1, B 2
      READ 6, 5 P5
   6. FORMAT (E 9.2)
   5. READ 60, B 1, C 1, B 2, CODE
  60. FORMAT (F 3.0, 2 F 6.4, 63 X, 12)
   7. READ 8, A, B, CL—
   8. FORMAT (2 F 5.2 68 X, F 2.0)
      IF (CL—1.) 88, 5, 5
  88. L = O
      H = B—A
      NX = 1
      T (1) =. 5* (F (A) + F (B))
      DO 10 K = 1, 10
      S = O
      X = H/2. + A
      DO 151 = 1, NX
      S = 2 + F (X)
  15. X = X + H
      FNX = NX
      FM = 2/5 NX
      T (K + 1) = . 5* (T (K) + FM)
      G = 1
      DO 20 J = 1, K
      G = 1
      DO 20 J = 1, K
      G = 4. *G
      1 = K — J + 1
  20. T (1) = T (1 + 1) + (T (1 + 1) — T (1)) / (G — 1.)
      EPS 1 = ABSF ((T (1) — T (2)) / T (1))* G
      IF (EPS 1—EPS) 16, 16, 4
   4. H = . 5* H
  10. NX = NX + NX
      L = 1
  16. R = T (1) * (B A)
      IF (SENSE EWITCH 1) 17, 18 —
  17. PRINT 11, CODE, B 1, C 1, B 2, R, 1.
  18. PUNCH 11, CODE, B 1, C 1, B 2, R, L
  11. FORMAT (5 X, 12, 45 15.4, 9 X, 12, 2 H 88)
      GO TO 7
      END
      FUNCTION (U)
      COMMON BL C 1, B 2
      F = B 1* (U** C 1)* EXPF (— B 2* U)
      RETURN
      END
```

REFERENCES

1. BENJAMIN, B.: Elements of vital statistics, London, George Allen and unwin LTD, 1959.

2. COX, PETER R.: Demography, Cambridge, The University Press, 1959.

3. NATIONAL CENTER For HEALTH STATISTICS: International Comparison of Perinatal and Infant Mortality U. S., and six European Countries, Washington, D. C., U. S. Department of Health, Education, and welfare, Series 3, No—6.

4. NATIONAL CENTER For HEALTH STATISTICS: Infant Mortality Problems In Norway, Washington, D. C., U. S. Department of Health, Education, and welfare, 1967, Series 3—No. 8.

5. UNITED NATIONS: Demographic Year Book for Years from 1957—1967, New York.

6. UNITED NATIONS: Population Bulletin, No. 2, 3, 6, New ork.

٧ — الجهاز المركزى للتعبئة العامة والاحصاء : تقييم بيانات المواليد ووفيات الرضع فى المناطق المختلفة بالجمهورية العربية المتحدة ، القاهرة ١٩٦٧ .

٨ — الجهاز المركزى للتعبئة العامة والاحصاء : زيادة السكان فى الجمهورية العربية المتحدة وتحدياتها للتنمية ، القاهرة ١٩٦٦ .

٩ — الجهاز المركزى للتعبئة العامة والاحصاء : احصاءات المواليد والوفيات للسنوات ١٩٦٢ ، ١٩٦٣ ، ١٩٦٤ ، ١٩٦٥ ١٩٦٦ ، القاهرة .

١٠ — الجهاز المركزى للتعبئة العامة والاحصاء : مجموعة الاحصاءات الحيوية للجمهورية العربية المتحدة من سنة ١٩٣٠ ، القاهرة ١٩٦٨ .

١١ — الجهاز المركزى للتعبئة العامة والاحصاء : التصنيف الدولى للامراض وأسباب الوفاة : المراجعة الثامنة لمنطقة الصحة العالمية ، القاهرة ١٩٦٩ .

١٢ — حسين عبد العزيز حلمى : دراسة مقارنة لنمط وفيات الرضع فى الجمهورية العربية المتحدة وبعض الدول المتقدمة . رسالة الماجستير مقدمة الى كلية الاقتصاد والعلوم السياسية جامعة القاهرة عام ١٩٧٠ .