A MODEL FOR HUMAN REPRODUCTION : THE CASE OF EGYPT

By

ATEF M. KHALIFA and HODA RACHAD

I. S. S. R., Cairo University

INTRODUCTION

It is not a new assertion that population growth is one of the main obstacles to social and economic development in the developing countries. Egypt by any means is no exception. Population projections showed an iminent and frightening prospect of growth beyond any economic plan to handle. Modern population growth occurs through a decline in the death rate while the birth rate remains at a relatively high level. The widening excess of births over deaths that results produces an accelerating increase of population. A second phase in the transition takes place when fertility enters on a decline and approaches once again the level of mortality. Egypt is lingering in the first phase of the transition, i.e., low mortality, high fertility, and rapid growth. It is not possible to examine the behavior of her population in the second phase through analysis of longitudinal or historical data. One of the alternatives is to employ simulation modeling approach.

In the present paper we present an extension and a modification of a model suggested by Tolba(1) few years ago. Our present modifications help acheive a more realistic model.

The broad purpose of this model is to obtain rapid generation, in an experimental way, the relationship between the controllable and uncontrollable variables. This demographic model attempts to give a simulation of the reproduction process of women during their child-bearing period to find out how the population may grow under different possible circumstances. In addition, it attempts to study the effect of each suggested factor and combinations of factors on the process of population growth.

(1) Sponserd by the Institute of Statistical Studies and Research Through on agreemat ,NO-NCHS-ARE 4. See : S. E. Tolba, on Human Reprodction I. S. S. R. monographic Series.

This human reproduction model is a deterministic, macrodynamic model. This means that it employs expected values involving numerical calculations of the number of events. It means also that probabilities operate on cohort of women in their childbearing ages. In addition, the model may be considered as dynamic within the cohort, since fecundability is assumed to vary according to several factors as will be seen.

Models of this kind have been developed by Henry (1961), sheps and Perrin (1963) and Potter and Sakoda (1966). The latter model developed by Potter and Sakoda (FERMOD) is similar to our present model. It is designed to follow a homogeneous marriage cohort of women till the end of their reproductive periods. The model operates with constant probabilities of events as well as constant durations of gestation and postpartum sterility. The only exception to this deterministic approach is the duration of postpartum sterility following live birth where a probability distribution is assumed.

The present model, as will be explained in detail, has been criticized in three ways :

1. All members of the cohort are assumed to survive throughout the simulated time-period(2).

2. The model did not take the effect of infant mortality into consideration.

3. It is assumed either-that all parameters are independent of age, or that all women are exactly the same age(3).

Our main objective in the present paper is to extend and modify the model to take two effects throughout the simulated time periods, to take care of the first two criticisms. They are :

1. The effect of maternal mortality.

2. The effect of infant mortality.

---

(2) E. Holmberg, *Fecundity, Fertility, and Family Planning, Demographic* Institute, Univ. of Gothenburg, Sweden, 1970.

(3) Ibid, p. 10.

THE MODEL

This model attempts to find out how population may grow under different circumstances. It is however, not a complete model as it only considers some of the factors that are thought to influence human reproduction for a cohort of women throughout their childmearing period.

In the model five separate states S_i $1 = 1, 2, 3, 4$) are distinguished : So fecundable state, S_1 pregnant state and three states (S_2-S_4) refering to postpartum sterility as follows :

S_2 -- postpartum sterile state following abortion,

S_3 -- postpartum sterile state following still birth, and,

S_4 -- postpartum sterile state following live birth.

Like all models, it requires a number of simplifying assumptions :

(a) Probability that a fertile fecundable non-pregnant woman conceives during a month is constant.

(b) Probability that a pregnancy will end in a postpartum period associated with abortion, still birth or live birth is constant.

(c) The time in months required by female from state of abortion, still birth or live birth to state of fecundability is constant.

Each woman in the cohort is assumed to be married and fecundable at the start and still married at the end of the reproductive period. The effect of divorce will be examined in a future paper. Each month, cohorts are brought forward in time for a fixed period according to the postulated probablities.

The model as suggested :

$$Y(R) = (1 - \rho)\, Y(R - 1) + \theta(2)\,.\rho\,.\, Y(R - B(2)) + \theta(3)\,.\rho\,.$$
$$Y(R - B(3)) + \theta(4)\,.\rho\,.\, Y(R - B(4)) \qquad (1)$$

with $Y(t) = 0$ ' if $t \leqslant 0$

and $\theta(2) + \theta(3) + \theta(4) = 1$ for $R \geqslant 10$

Number of live births during month $R = \theta(4)\,.\,\rho\,.\,Y(R-10)$ (2)

Where :

$Y(R)$ = number of females of the cohort in nonpregnant fecundable state at time R.

B(i) = time from the beginning of the month in which conception leading to S_i occurs, up to the moment of entering nonpregnant fecundable state again for the first time (i = 2, 3, 4).

θ (i) = probability that a given pregnancy will end in a sate i (i = 2, 3, 4).

ρ = Probability, at the beginning of a monthly cycle of a certain female, that this female will conceive during the month. Two programs were written.

Program I : Computes for each set of suitable values of the parameters considered, the number of monthly births for a cohort of married fertile women. It gives a measure for the effect of an increase in the value of B (4), or a decrease in the value θ (2), on the number of live births during 5, 10 or 15 years.

The computations have been carried out for each of 140 sets of values of the parameters. These values are as follows :

(1) ρ is given, in succession, the values :
.300, .250, .200, .150, .100, .050, .010.

(2) With each value of ρ, θ (2) is given, in succession, the values :
.10, .20, .30, .40.

(3) θ (3) is taken equal to zero throughout the computltions.

(4) B(2) is taken equal to 5 months throughout the computations.

(5) B(3) is taken equal to 12 months throughout the computations.

(6) With each set of values of ρ and θ (2), B(4) is given the values

12, 15, 18, 21, 24, 27 months.

Program II : Computes for each set of suitable values of the parameters considered, the number of live births in each of 15 consecutive years. It gives a measure of the effect of a change in the value of ρ on the number of births in a given period. Computations have been carried out for each of 672 sets of values of the factors. These sets are as follows :

(1) B(2) is given in succession, the values : 3, 4, 5, 6 months.

(2) With each value of B(2), is given, in succession, the values,
.300, .250, .200, .200, .150, .100, .050, .010.

(3) With each set of values of B(2) and ρ, B(4) is given, in succession, the values : 12, 15, 18, 21, 24, 27, months.

(4) With each set of values of B(2). ρ and B(4), θ (2) is given, in succession, the values : .10, .20, .30, .40.

The results of the two programs described above may be used to give recommendations as regards keeping population growth under control.

It may be desired to have the number of births decreased by a certain percentage. To attain this, one or both of the following ways may be contemplated :

(i) To use effective contraceptive measure which would increase the value of β (4) and thus increase the duration of sterility after a live birth.

(ii) To use some less effective contraceptive measure which would decrease the value of ρ ,i. e., the probability of conception. Clearly either measures should be taken only by women of high fertility, i. e. where :

— β (4) is less than a certain duration. (and/or).

— ρ is greater than a certain value.

So we shall consider cases where β (4) $\leqslant$ 18 and ρ $\geqslant$. 150

Tabulations of program I show the following ; that if β (4) $\leqslant$ 18 and $\rho \geqslant$ .15, an increase in the value of β (4) will give a percentage decease in the number of live births during a sufficient long period as shown approximately in the following table :

Increase in β (5)	Decrease in live births
6 months	From 12.5% to 24.5%
9 months	From 16% to 32%
12 months	From 22% to 38%

The bigger decrease being associated with big values of and small values of 0 (2), i. e., with women of high fertility.

Tabulations of program II show that :

1. If $\rho \geqslant$ 200 and β (4) $\leqslant$ 18, a small decrease in the value of ρ would

not be very effective. For example if ρ is decreased so that it now becomes $p = .050$, the number of live births will be decreased by a value between 4% and 10%, the bigger decrease being associated with big values of θ (2), i. e., with women of comparatively small fertility.

2. If $\rho \geqslant .250$ and β (4) $\leqslant 18$, then to decrease the number of live births by 50% through changing ρ only, it would be necessary to take a measure that would make $\rho \leqslant .050$.

3. If β (4) $\leqslant 18$, then a measure that would decrease ρ from .200 or more to .010 would decrease the number of live births by over 80%.

MODIFICATIONS OF THE MODEL

In order to get a more realistic model we proceed to consider :

(a) The effect of infant mortality.

(b) The effect of maternal mortality.

First : *The effect of Infant Mortality :*

Infant mortality's effects can operate in two ways : by affecting number of children alive and fecundability of mothers :

(i) *Effect of Infant mortality on the number of children alive :*

We start by the assumption that infant mortality affects only the number of children alive and that it has no effect on women fecundability. In this case the number of children that remain alive for a given period is no longer an accumulation of births (for a cohort of women) in the period considered. We have to consider mortality probabilities by months. Let us define q as the probability for a child at age zero to stay alive for t months, and v_R as the number of live births during month $R (R \geqslant 10)$, we may say that number of children who stayed alive in that period = period $\sum_{R=10}^{period} V_K$ q (period-R) (3) i. e. number of live birth during month R who stayed alive for the whole period =
θ (4). ρ Y(R-10). q(period-R) (4)

To measure the effect of an increase in the value of B(4), or a decrease in the value of θ (2) on the number of live births during 5, 10 or 15 years, taking into consideration infant mortality a third program was computed using these equations. Y(0) is taken to equal one million.

Same sets of values used earlier were reassigned for the parameters. Using the male national life table for Egypt 1960, the following values of q(i) are assigned. We assume that the relative distribution of deaths over the twelve months of the year remains constant, which is not very accurate especially for infantile deaths (below one year of age). However, these values may be easily reassigned if the required data are available. The assigned q(i)'s values are as follows :

q(i) = .892	i =	1, ...,	12
= .833	=	13, ...,	24
= .802	=	25, ...,	36
= .789	=	37, ...,	48
= .781	=	49, ...,	60
= .775	=	61, ...,	72
= .771	=	73, ...,	84
= .767	=	85, ...,	96
= .764	=	97, ...,	108
= .761	=	109, ...,	120
= .758	=	121, ...,	132
= .755	=	133, ...,	144
= .753	=	145, ...,	156
= .751	=	157, ...,	168
= .748	=	169, 170	

Results of the third programs shows that introducing infant mortality makes quite a difference in the number of accumulated births in a period. Though, general conclusions derived from the first program are almost the same as those derived from the third program.

For example : the percentage decrease in number of live births in 5 years if β (4) is increased from 12 to 18 month, θ (2) remaining equal to .10, R0 = 300, β (2) = 3, is 23.4% and 23.5% in the first and third programs respectively.

To measure the effect of infant mortality on the number of live births in each 15 consecutive years which gives a measure of the effect of a change in the value of ρ on the number of birth in a given period, a fourth program was computed.

The same sets of values used in the second program for the parmeters and q(i) were given the same values as in the third program. Results of the fourth program show that taking infant mortality into consideration makes a great difference in the number of accumulated births in the period. The general conclusions derived from each program are different, for example the percentage decrease in the number of five births in 15 years if ρ is decreased from 0.30 to 0.25, and (2) remaining equal to .10, β (2) = 3, β (4) = 12 ; is 4.8% and 8.8% in the second and fourth programs, respectively.

(ii) *The effect of infant mortality on the number of children alive and on the fecundability of mothers.*

Now we will consider effect of infant mortality on both number of children alive and on women's fecundability.

The fifth and sixth programs are written to show this effect. To study the effect on fecundability we defined another state s(5) = postpartum sterile state following live birth that has died in first month. $\beta(5)$ = time from the beginning of the month in which conception leading to state (5) occurs, up to the moment of entering nonpregnant fecundable state again for the first time. In this case we will have a probability of moving from s(4) to s(5), this probability = u_1 = (1-q (1)). It follows, that if a woman stays in s(4), then the relation is unaltered. Furthermore, it is to be noted that we could have defined several states, for example :

s(5) : postpartum sterile state following a live birth that has died in the first month.

s(6) : postpartum sterile state following a live birth that has died in the second month.

s(7) : postpartum sterile state following a live birth that has died in the third month.

if β (4) = 9 + x months, we take s(i) up to x months, for sake of simplicity, we took only one case, that is s(5).

β (5) is β (4) generally.

β (5) is assumed to be less by one month, hence the relation beccomes :

$$Y(R) = (1-\rho).\ Y(R-1) + \theta(2).\ \rho.\ Y(R-\beta(2)) + \theta(3).\rho.$$
$$Y(R-\beta(3)) + (1-U_1)\,\theta(4).\ \rho.\ Y(R-\beta(4)) + U_1\theta(4).\ \rho.\ Y(R-\beta(5)).$$

where $Y(t) = 0$ if $t \leqslant 0$

and $\theta(2) + \theta(3) + \theta(4) = 1$ for $R \geqslant 10$.

The number of live birhs during month R who stayed alive for the whole period = θ (4). ρ Y(R — 10) q (period-R), using equations (5) and (6) the fifth and sixth programs are written. The fifth program computes the number of monthly births for a cohort of married fertile women and it gives a measure of the effect of any increase in the value of β (4) or any decrease in the value of θ (2), on the number of live births during 5, 10 or 15 years taking into consideration the effect of infant mortality on both number of children alive, and women fecundability.

The same sets of values used in the third program were assigned for the parameters ; and β (5) was given in succession the values : 11, 14, 17, 20, 23, 26 months, which are always one month less than the values given to β (4) : (12, 15, 18, 21, 24, 27, months).

Comparing the results of programs 3 and 5, we see that the accumulated births for the cohort of women in the given period are less, but the general conclusions derived from program 5 are almost the same as in program 3. For example : The percentage decrease in number of live births in 5 years if β (4) is increased form 15 to 18 months, θ (2) remaining equal to .10, was 9.1% in the third and fifth program respectively. A further example : the percentage increase in number of live birth in 5 years if θ (2) is decreased from .20 to .10, B(4) remaining equal to 12 months was 6.8% and 7.3% in the third and fifth program respectively.

We notice, though, that the percentage decrease in the number of accumulated births in the given period due to an increase in the value of β (4) tends to increase in the fifth program with comparison to the third program. This makes the results of the fifth program almost identical to the results of the first program. For example : the percentage decrease in number of live births in 5 years if β (4) is increased from 15 to 18 months, and θ (2) remained equal to .10 in the first programs, was equal to 9.4% (note that this value is equal to 9.3% in the fifth program).

In addition, it should be noted that the percentage increase in the number of accumulated births in a given period due to a decrease in the value of θ (2) tends to increase in the fifth program in comparison to to the third program. This makes the results of the fifth program differ greatly from the results of the first program. For example : the percentage increase in number of live births in 5 years if θ (2) is decreased from .20 to .10, β (4) remaining equal to 12 months in the first program, is equal to 5.2% (note that this value is equal to 7.3% in the fifth program, so we see there is a great difference).

In sum, our conclusions of this part are as follows : If our aim is to measure the percentage decrease in the number of accumulated births in a given period due to an increase in the value of θ (4) we can use the first program. If our aim is to measure the effect of changing the value of β (2) on the number of births, the fifth program is more appropriate. Furthermore, if we mean to compute the number of accumulated births in a given period we may use either the second or third program although available data will be a factor in the decision of using either model.

The sixth program computes the number of live births in a given period and gives a measure for the degree of the effect of a change in the value of ρ on the number of births in this period, taking into consideration the effect of infant mortality on both number of children alive and women's fecundability.

The same sets of values used in the fourth program were assigned for the parameter, and β (5) was given the same values as in the fifth program.

Comparing the results computed by the second, fourth and sixth programs it can be seen that though infant mortality made quite a difference in the number of accumulated births in a period, the general conclusions derived from these three programs are about the same. For example : the percentage decrease in the number of live births in 15 years if ρ is decreased from 0.30 to 0.25, and θ (2) remains the same, is equal to 4.6% in the three programs.

Thus, if our objective is to measure the percentage decrease in the number of accumulated birth due to a change in the value of ρ, we may use the first program, but if our main objective is to compute the number of accumulated births in a given period we may use either the second or the third program.

Second : *The Effect of Maternal Mortality :*

To consider the effect of maternal mortality we introduced a new parameter to the equation. Which is the probability for a female at age x to live for k months. We assumed that we started initially with a cohort of the same age which was taken equal to 20. The equation becomes :

$$Y(R) = (1-\rho).\ Y(R-1).\ PM(R-1,1) - \theta(2).\ \rho.\ Y(r-\beta)(2)).\ PM(R-\beta(2),$$

$$\beta(2)) + \theta(3).\ \rho.\ Y(R-\beta(3)).\ PM(R-\beta(3),\ \beta(3)) - (1-u_1).\ \theta(4).$$

$$Y(R-\beta(4))\ PM(R-\beta(4)) - u_1.\ \theta(4).\rho.\ Y(R-\beta(5)).\ PM(R-\beta(5),\ \beta(5))$$

Where $Y(t) = 0$ if $t \leqslant 0$

$\theta(2) + \theta(3) + \theta(4) = 1$ for $R \geqslant 10$

number of live births during month R who stayed live for the whole period $= \theta(4).\ \rho\ Y(R\text{-}10).\ q(\text{period-R})$. where,
PM(z, k) = probability for a female at age 20 + z months to live for k months. The seventh program measures the effect of any increase in the value of β (4), or any decrease in the value of θ (2) on the number of live births during 5, 10 or 15 years taking into consideration infant and maternal mortality. The same sets of values of the parameters as used in program 1 were given. For PM(z, k), we assigned the following values :

PM(I, 1)	= 0.998	i = 0,73
PM(I, 1)	= 0.997	i = 74,180
PM(I, β 2(L))	= 0.998	i = 0,73
PM(I, β 2(L))	= 0.997	i = 74,180
for L	= 1, 2, 3, 4.	
PM(I, β 4(1))	= 0.998	i = 0,73
	= 0.997	i = 74,180
PM(I, β 4(k))	= 0.997	i = 0,73
	= 0.995	i = 74,85
	= 0.994	i = 86,180
for k	= 2, 3, 4, 5.	
PM(I, β 4(6))	= 0.995	i = 0,67

	$= 0.994$	$i = 62,74$
	$= 0.989$	$i = 75,87$
	$= 0.997$	$i = 88,171$
	$= 0.990$	$i = 172,180$
$PM(I, \beta 5(1))$	$= 0.998$	$i = 0,73$
	$= 0.997$	$i = 74,180$
$PM(I, \beta 5(k))$	$= 0.997$	$i = 0,73$
	$= 0.995$	$i = 74,85$
	$= 0.994$	$i = 86,180$
for k	$= 2, 3, 4, 5.$	
$PM(I, \beta 4(6))$	$= .995$	$i = 0,61$
	$= 0.994$	$i = 62,74$
	$= 0.989$	$i = 75,87$
	$= 0.997$	$i = 88,171$
	$= 0.990$	$i = 172,180$

These values were computed using the female national life table for the Arab Republic of Egypt (1960), assuming that the relative distribution of deaths over the twelve months of the year remains constant. This is an explanation of why $PM(I, \beta 4(k))$ for $k = 2, 3, 4, 5$ are equal since $\beta\ 4(2) = 15$, $\beta\ 4(3) = 18$, $\beta 4(4) = 21$; $\beta\ 4(5) = 24$ which are all within the second year.

Comparing the results computed by the seventh and first programs we see that the accumulated births for the cohort of women are less in program 7 than 1, but the general conclusions derived from the first are almost the same conclusions derived from the seventh program. This is quite a significant result because if our aim is to measure the effect of a change in the value of $\theta\ (2)$ on the number of live births the first program is cuitable. Of course, if we mean to compute the number of accumulated births in a given period, it will be more appropriate to use program 7, which takes into consideration infant and maternal mortality.

The eighth and last program measures the effect of a change in the value of ρ on the number of births in this period, taking into consideration the effect of infant and maternal mortality. The same sets of values used in the sixth program were given.

Comparing the results computed by the eighth & second programs, we note that the conclusions derived from the eighth program are quite different from those derived from the second. For example : The percentage decrease in the number of live births in 15 years, if β is decreased from 0.30 to 0.25, and 0 (2) remains the same, is equal to 4.8 and 8.6 in the second and eighth programs respectively.

CONCLUSION

Finaly we summarize some conclusions concerning the effect of changing the values of the parameters on population growth. These conclusions are deduced from the tables computed and only an illustration is given here.

(1) The effect of increasing the value of β (4)

(4) is defined as the time from the beginning of the month in which conception leading to a live birth, up to the moment of entering nonpregnant fecundable state again for the first time. Let us consider :

$$.150 < \rho < .300$$

$$.10 < \theta(2) < .40$$

$$3 < \beta(2) < 6$$

$$12 < \beta(4) < 27$$

TABLE 1

The effect of increasing the value of β (4) taking the effect of infant and maternal mortality.

Increase of β (4) by :	Decrease in the number of births in 15 years	Remarks
6 months	between 13.3 and 27.0%	the percentage decrease
9 months	between 20.5 and 35.2%	in the number of birth is

12 months	between 29.8 and 41.6%	bigger wherever ρ is
15 months	between 34.8 and 48.2%	bigger ,$\theta(2)$ is Smaller,
		$\beta(2)$ is smaller.

These results are deduced from the programs that take into consideration the effects of infant and maternal mortality.

Comparing these results with those obtained from the simplest program I given in table (2), we notice that there are very small difference :

TABLE 2

The effect of increasing the value of $\beta(4)$ using the simple program I

Increase of $\beta(4)$ by	Decrease in the number of births in 15 years
6 months	between 13.6 and 27.45%
9 months	between 20.6 and 35.95%
12 months	between 27.7 and 42.45%
15 months	between 34.9 and 48.75%

(2) The effect of decreasing the probability of abortior θ (2) :
If we consider :

$$.150 < \rho < .40$$

$$.20 < \theta(2) < .40$$

$$3 < \beta(2) < 6$$

$$12 < \beta(4) < 18$$

Taking into consideration the effects of infant and maternal mortality we can deduce the following table (3) :

TABLE 3

The effect of decreasing the probability of abortion

The intial value θ (2)	The new value θ (2)	The increase in number of births in 15 years	Remarks
.20	.10	between 4.1 and 8.8%	The percentage increase in the number of births increases whenever : ρ decreases, β (2) increases, the initial value of θ (2) is bigger.
.30	.20	between 5.1 and 9.9%	
.40	.30	between 6.5 and 12.6%	

Comparing these results with those deduced from the simple program and only for a period of 5 years (given in table (4)), we notice that there are little differences :

TABLE 4

The effect of decreasing the probability of abortion using the simple program

θ (2) The initial value	θ (2) The new value	The increase in number of birth in 5 years
.20	.10	between 4.7 and 8.95%
.30	.20	between 5.9 and 10.5%
.46	.30	between 7.2 and 12.75%

(3) The effect of decreasing the probability of conception (ρ) :
If we consider the following values :

$.150 < \rho < .250$

$.10 < \theta(2) < .40$

$\beta(2) = 6$

$12 < \beta(4) < 27$

Then, the effect of decreasing ρ can be summarized in the following table :

TABLE 5

The effect of decreasing (p)

The initial value ρ	The new value ρ	The decrease in the number of Births in 15 years	Remarks
between 0.15 an d 0.25	0.05	between 45.7 and 66.3%	The percentage decrease in number of births is bigger
	0.01	between 91.9 and 85.6%	Whenever : θ (2) is Smaller β (4) is Smaller θ (2) increases.

In general, the absolute numbers of accumulated births differ greatly when the effects of infant and maternal mortality are being considered. However, the percentage changes in the number of births did not significantly differ among the programs. This was expected since we consider the relative rather the absolute changes.

REFERENCES

1. TOLBA, S. E., On Human Reproduction, Institute of Statistical Studies and Research, memographic series.

2. SHEPS, M.C. and PERRIN E. B. 1963. changes in birth rates as a function of contraceptive effectiveness. Some application of stochastic models. I. *Am. public assoc.* 53 : 1031.

3. SHEPS, M.C., and PERRIN, E. B. 1963. The distribution of birth internals under a class of stochastic fertility models, Firth Intern. *Biometric conference.*

4. E. HOLMBERG, *Fecundity, Fertility, and Family Planning,* Demographic Institute, Univ. of Gothenburg, Sweden, 1970.