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A Probabilistic Model For Studying Factors Affecting Female Employment

Amina Esmail Abu - Hussien
Associate Professor of statistics
Faculty of commerce
Al-Azhar University , Girls Branch

Abstract

Female employment is a complex phenomenon which is affected by many social, economic and demographic factors. Thus, the traditional methods used in studying female employment make it very difficult to evaluate the effects of these interrelated factors.

This paper is concerned with the development of a probabilistic model for studying the effects of these factors. A brief analysis to the most important factors affecting female employment is illustrated. A general probabilistic model for studying female employment is developed. A special case of the suggested general probabilistic model is introduced. An application to the special case model using published data of the most recent population census conducted in A.R.E. in 1986 is represented.

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1- Introduction :

The economic and social development of any country is influenced by the number of employed and their qualifications.Labour force supply is influenced mainly, among other factors, by the population age and sex composition.

Factors affecting the employment status of both males and females differ greatly, so to understandfully how the work force of both males and females affect the total employment of a country; separate investigation to the participation rates by sex is necessary.

It is noticed that, male lobour force participation rates are higher and more stable than the corresponding rates for females. Male labour force participation rates by age groups in almost all nations have the same shape where the rates start from a low level at the initial age of entry and reach its peak and remains at this high level through the central working ages and then begins to decline sharply[2]. On the other hand, there are great discrepancies in female labour force participation rates by age groups. Such discrepancies may be explained by the great number of social, economic and demographic factors affecting female employment . Hence, female labour force supply is the most important factor in the changes, of the total manpower supply. This fact is very useful for manpower planners and policy makers.

The literature is rich in studies concerning with female employment . The majority of these studies are descriptive studies. The existing mathematical models deal with female employment as a function of age only. These models do not take into consideration the other social, economic and demographic variables which have a great effect on female employment. Thus, it is of great importance to develop a mathematical model to study the main social, economic and demographic variables affecting female employment.

In the present study an attempt is made to develop a probabilistic model for studying the main social, economic and demographic variables affecting female employment. The resulting probability distributions provide the researcher with statistical techniques that enables him to make statistical inference about female employment. Also, these probability distributions are very useful in simulation studies in which it is required to assign to each input variable the corresponding probability distribution.

Section two is devoted to investigate the main social, economic and demographic determinants of female employment. A general probabilistic model for female employment is developed in section three. A special case of the suggested general probabilistic model for female employment is introduced in section four. An application to the special case model using published data of the most recent population census conducted in A.R.E. in 1986 is given in section five.

2. The Most Important Factors Affecting Female Employment.

Female employment is influenced largely by a great number of social, economic and demographic factors. The main factors which have been chosen, in the present study, to be the determinants of female employment include; age, marital status, educational status, parity, age of children, family income and place of residence. In the present section a brief analysis is made to throw some light on these factors.

(1) Age :

It is expected that female labour force participation rates reach a peak before the age of marriage and childbearing, then the rates tend to decline as a result of increasing children responsibilities which may cause withdrawal from the labour force.

(2) Marital Status :

It is expected that female labour force participation rates for married females are much lower than those for single females and the lowest rates may be found among ever married women with children under school age irrespective of their present marital status. However, those mothers without a husband present, whether because of widowhood or divorce are more likely to be in the labour force because of their economic need and the same situation is expected for the other ever married females with older children, where females tend to re-enter the labour force as children attend school [7]. In some situations, the participation rates for married females are higher than those for unmarried females which is against the common pattern. This may reflect a need to contribute to family finances.

(3) Educational Status :

Increasing educational levels have created a desire for females to participate in the labour force [7]. It is expected that a positive relationship is found between educational status and female labour force participation rates. This relation tend to be the reverse in countries with high levels of unemployment rates.

(4) Parity :

It is expected that parity (number of live births a woman has had) is inversely related to female labour force participation rates because of increasing household work and caring for children. However, increasing parity may affect female labour force participation rates in an opposite direction as a result of economic need.

(5) Age of Children :

Parity may have a little effect on female labour force participation rates compared with age of children. It is expected that the participation rates for females with children under school age is much lower than the corresponding rates for females with older children.

(6) Family Income :

It is expected that a negative relationship is found between family income and female labour force participation rates.

(7) Place of Residence :

The above relationships varies significantly between urban and rural areas. Thus, it is of great importance in studying these relationships to distinguish between urban and rural areas or between developed and less developed countries.

The previous analysis reveals that, female employment is a complex phenomenon which is affected by many social, economic and demographic factors. Thus, the traditional methods used in studying female employment make it very difficult to evaluate the effects of these interrelated factors . A general probabilistic model for studying female employment is developed in the present study . It can provide the researcher with a statistical technique that enables him to test several assumptions about this phenomenon. Also, the resulting probability distributions may be used in simulation models which allow for considering more factors under less restrictive assumptions than in the case of analytical models [5].

3. A General Probabilistic Model For Female Employment

The discussion in section two reveals that female employment is affected by many factors. Each factor affect female employment with different probabilities. If equal probabilities of being employed are given to females regardless of the variations in each factor, then misleading results will occur. Hence, it is of great importance to take the differential effects in each factor into consideration in developing a model for female employment. To achieve this aim, a trial is made in the present section to derive the probabilities of female employment as mathematical functions of the most important factors affecting it. In the suggested model, a set of univariate probability density functions (p.d. f.s) are developed. Each p.d.f. relates the probability of female employment to only one of the factors affecting it.

Sub-section (3.1) introduces the assumptions of the model and in sub-section (3.2), the model formulation is given.

3.1 Model Assumptions

In the present model it is assumed that :

1) We have a female population in which employment status is affected by K factors.
2) Factors affecting female employment are represented by continuous random variables.

3.2 Model Formulation

The probabilistic approach developed here depends on the following well known relation between the p.d.f. of any variable and the force of happening of this variable.

$$\phi(x) = \frac{f(x)}{1-F(x)} = \frac{f(x)}{S(x)} \qquad (3.1)$$

Where $\phi(x)$ is the force of happening, $F(x)$ is the probability distribution function and $S(x) = 1-F(x)$ is the survival distribution function[3].

For the development of our model, we first introduce the definitions of the forces of female employment and the method of estimating these forces. Then, the corresponding survival distribution functions are given. Finally, the resulting p.d.f.'s are introduced.

Let $\phi_i\ (x_i;\ \theta_i)$ be the crude force of female employment for the i-th variable with parameter vector θ_i i,e $\phi_i\ (x_i;\ \theta_i)$ describes the instantaneous probability of having a job due to the i-th variable in a small interval $(x_i;\ x_i+\Delta x_i)$ in the presence of all other factors in the population.

The form of the function $\phi_i\ (x_i;\ \theta_i\ )$tells us the relation between the i-th variable and the chance of having a job. Large values of $\phi_i\ (x_i;\ \theta_i)$ indicate the values of the i-th variable at which the chance of having a job is high and such information is very useful to the manpower planner.

To estimate the observed pattern of the crude force of female employment for the i-th variable, $\phi_i\ (x_i;\ \theta_i\ )$, the central female employment rates $[\hat{\phi}_i\ (x_i)]$ are calculated for each group of the i-th variable as follows :-

$$\hat{\phi}_i\ (x_i) = \frac{\text{number of females who have a job in any specified group of the i-th variable}}{\text{number of females at risk of having a job in the same group of the i-th variable}} \qquad i = 1,2,\ ....K. \qquad (3.2)$$

the numerator and the denomenator of equation (3.2) must belong to the same population and the same period of time.

$\hat{\phi}_i(x_i)$ are referred to as the instantaneous discrete probabilities of female employment. Having a job is a process which can occur at any time in a person's career and so it is reasonable to treat it as a continuous variable[1]. It is assumed that $\phi_i(x_i; \theta_i)$ are continuous functions for $x_i \geq 0$.

In order to specify a mathematical function that describes $\phi_i(x_i;\theta_i)$, the corresponding observed central rates, $\hat{\phi}_i(x_i)$, are used in drawing the observed curve from which the theoretical curve can be obtained.

To each one of the crude forces of female employment, $\phi_i(x_i; \theta_i)$, there corresponds a survival distribution function (s.d.f.) defined by :

$$S_i(x_i;\theta_i) = \exp.\left(-\int_0^{x_i} \phi_i(t;\theta_i)dt\right) \quad i=1,2........K \qquad (3.3)$$

The estimated survival functions immediately give estimates to the corresponding p.d. f. s, $f_i(x_i;\theta_i)$ as follows.

$$f_i(x_i;\theta_i) = \phi_i(x_i;\theta_i)\exp.\left(-\int_0^{x_i} \phi_i(t;\theta_i)dt\right) \qquad (3.4)$$

$$= \phi_i(x_i;\theta_i).S_i(x_i;\theta_i). \qquad i=1,2......... K$$

Equation (3.4) gives the p.d.f.s. of female employment as functions of only one of the variables affecting it. These probability distributions are very useful in simulation studies in which it is required to assign to each input variable the corresponding probability distribution. Also, these probability distributions provide the researcher with statistical techniques that enables him to make statistical inference about female employment.

4. Special Case Of The General Probabilistic Model For Female Employment.

In this special case, it is assumed that the most important factors affecting female employment are age and educational status. Thus, we have the following two forces of female employment :-

$\phi_1(x_1;\theta_1)$ is the crude force of female employment as a function of age, x_1, with parameter vector θ_1, $\phi_2(x_2;\theta_2)$ is the crude force of female employment as a function of educational status , x_2, with parameter vector θ_2.

$\phi_1(x_1;\theta_1)$ describes the relation between age, x_1, and the chance of having a job. The chosen measure to estimate a mathematical function that describes $\phi_1(x_1;\theta_1)$ in the present study is the female age - specific employment rate. This rate in a certain age group (eg. the j-th age group) may be defined using equation (3.2) as follows:

$$\hat{\phi}_1(x_1) = \frac{\text{number of females who are currently employed in the labour market in the j-th age group}}{\text{number of females at the mid point of the j-th age group}} \quad (4.1)$$

Data required for the calculation of female age -specific employment rates is the distribution of females by age and employment status.

$\phi_2(x_2\;;\theta_2)$ describes the relation between educational status, x_2, and the chance of having a job. To derive a mathematical formula for $\phi_2(x_2;\theta_2\;)$, a suitable measure for educational status is first introduced. The what might be called the female educational - specific employment rate seems to be a suitable measure. This rate in a certain educational level (eg. the l-th educational level) may be defined using equation (3.2) as follows:

$$\hat{\phi}_2(x_2) = \frac{\text{number of females who are currently employed in the labour market in the l-th educational level}}{\text{number of females at the mid point of the l-th educational level.}} \qquad .(4.2)$$

Data required for the calculation of female educational specific employment rates is the distribution of females by educational level and employment status.

Having determined the mathematical forms of $\phi_1\,(x_1;\theta_1)$ and $\phi_2(x_2;\theta_2)$, the corresponding s.d.f.s and p.d.f.s can be derived using equations (3.3) and (3.4) respectively as follows :

$$S_i(x_i;\theta_i) = \exp.\left(-\int_0^{x_i} \phi_i(t;\theta_i)dt\right) \quad i = 1,2 \qquad (4.3)$$

$$f_i(x_i;\theta_i) = \phi_i(x_i;\theta_i).S_i(x_i;\theta_i) \quad i = 1,2 \qquad (4.4)$$

5. Application

This section is concerned with an application to the special case of the general probabilistic model for female employment suggested in section four. Published data of the most recent population census conducted in A.R.E. in 1986 is used.

In this application it is assumed that :

1- Working age is taken to be the age interval (15-65) years.

2- The number of females who are currently employed in the labour market include; own account workers, paid workers and unpaid workers.

3- The number of females is not given at the mid year, since the census of A.R.E. in 1986 was carried out in November, however, it is assumed that it represents the corresponding number at the mid year.

4- Separate investigation is necessary for urban and rural areas to represent the effect of urban - rural differentials. Urban areas are considered only in the present study.

We first introduce the derivation of mathematical expressions for the crude forces of female employment $\phi_1(x_1;\theta_1)$ and $\phi_2(x_2;\theta_2)$ in sub - sections (5.1) and (5.2) respectively. Then, the corresponding survival distribution functions and probability density functions are derived in sub-section (5.3).

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5.1 Derivation of a mathematical expression for $\phi_1\ (x_1;\ \theta_1)$

To derive a mathematical formula for the crude force of female employment as a function of age, female age-specific employment rates are used in drawing the observed curve from which the theoretical curve can be obtained.

Using data for urban areas of A.R.E. in 1986, female age-specific employment rates are calculated as follows :-

1) Number of females in the not stated age interval have been redistributed among the age intervals according to their relative distribution.
2) Female age - specific employment rates are calculated using equation (4.1).

Calculations are shown in table (1) in the Appendix :

Graphical presentation of the relation between female's age and female age - specific employment rates is shown in figure (1) in the Appendix. This relation reveals that the shape for the force of female employment by age rises with age up to the mid point of the age interval (25 -30), then it declines gradually to retirement age. The investigation of this shape showed that it may be well described using Pearson's Type I system of frequency curves given by[4]:

$$\phi_1(x_1;\theta_1) = \phi_0\left(1+\frac{x_1}{a_1}\right)^{m_1}\left(1-\frac{x_1}{a_2}\right)^{m_2} \quad -a_1 < x_1 < a_2 \qquad (5.1)$$

where : $\theta_1 = [\phi_o\ ,\ a_1, a_2, m_1, m_2]$ & $m_1/a_1 = m_2/a_2$ and origin at the mode.

and the estimated force is :

$$\hat{\phi}_1(x_1;\theta_1) = 0.2185\left(1+\frac{x_1}{2.1933}\right)^{0.6393}\left(1-\frac{x_1}{8.7974}\right)^{2.5643} \qquad (5.2)$$

$$-1.4177 < x_1 < 5.5823$$

with origin at age 34.5883 (which is the modal value of x_1). Calculations are shown in table (3) in the Appendix

5.2 Derivation of a mathematical expression for ϕ_2 (x_2 ; θ_2) :-

To derive a mathematical formula for the crude force of female employment as a function of educational status, female educational - specific employment rates are used in drawing the observed curve from which the theoretical curve can be obtained.

Using data for urban areas of A.R.E. in 1986, female educational - specific employment rates are calculated as follows :

1) Number of females in the not stated educational level have been redistributed among the educational levels according to their relative distribution.
2) Female educational - specific employment rates are calculated using equation (4.2).

Calculations are shown in table (2) in the Appendix.

To draw the relation between female's educational status and female educational - specific employment rates, the following scores are given for each educational level : 0 for illiterate & 1 for read and write & 2 for primary certificate & 3 for less than medium certificate & 4 for medium certificate & 5 for more than medium certificate & 6 for university degree and equivelants & 7 for post graduate diploma & 8 for master's degree and 9 for doctor's degree.

Graphical presentation of the relation between female's educational status and female eductional - specific employment rates is shown in figure (2) in the Appendix . This relation is a discrete increasing function. it can be approximated by the following exponential formula:

$$\phi_2(x_2;\theta_2) = \alpha \exp. (\beta x_2) \qquad x_2 \geq 0 , \beta>0 \qquad (5.3)$$

where : $\theta_2[\alpha,\beta]$

and the estimated force is :

$$\hat{\phi}_2 (x_2;\theta_2) = 0.0223 e^{0.4903 x_2} \qquad x_2 \geq 0 \qquad (5.4)$$

Calculations are made using a manual scientific calculator.

Kolmogorov - Smirnov test is applied to test the goodness of fit for the data graduated by $\hat{\phi}_1 (x_1 ; \theta_1)$ and $\hat{\phi}_2 (x_2;\theta_2)$. It is found that it fits the data fairly well [6]

The resulting mathematical expressions for $\phi_1(x_1, \theta_1)$ and $\phi_2 (x_2,\theta_2)$ may throw some light on the general shape and the behaviour of these forces for any similar human population.

5.3 Derivation of the survival distribution functions and the probability density functions :-

Having determined mathematical expressions for $\phi_1 (x_1;\theta_1)$ and $\phi_2(x_2;\theta_2)$, the corresponding survival distribution functions can be derived using equations (4.3), (5.1) and (5.3) as follows :

$$S_1(x_1;\theta_1) = \exp.\left(-\int_0^{x_1} \phi_0(1+\tfrac{t}{a_1})^{m_1}(1-\tfrac{t}{a_2})^{m_2}\,dt\right) \quad (5.5)$$

$$S_2(x_2;\theta_2) = \exp.\left(-\int_0^{x_2} \alpha e^{\beta t}\,dt\right) \quad (5.6)$$

Then the probability density function of female employment as a function of age can be derived using equations (4.4), (5.1) and (5.5) as follows :

$$f_1(x_1;\theta_1) = \phi_0(1+\tfrac{x_1}{a_1})^{m_1}(1-\tfrac{x_1}{a_2})^{m_2}\exp.\left(-\int_0^{x_1} \phi_0(1+\tfrac{t}{a_1})^{m_1}(1-\tfrac{t}{a_2})^{m_2}\,dt\right) \quad (5.7)$$

where : origin at mode & $m_1/a_1 = m_2 / a_2$ & $-a_1 < x_1 < a_2$

and the probability density function of female employment as a function of educational status can be derived using equations (4.4), (5.3) and (5.6) as follows:

$$f_2(x_2;\theta_2) = \alpha e^{\beta x_2}\exp.\left(-\int_0^{x_2} \alpha e^{\beta t}\,dt\right) \quad x_2 \geq 0, \beta > 0 \quad (5.8)$$

The corresponding estimated survival distribution functions and probability density functions can be derived using the estimated forces.

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Appendix

Table (1)

Observed Female Age-Specific Employment Rates
For Urban Areas of A.R.E. in 1986

Age groups x_1 (1)*	Number of females (2)*	Corrected number of females (3)	Number of currently employed females (4)*	Corrected number of currently employed females (5)	$\hat{\phi}_1(x_1)=(5)\div(3)$ (6)
15-	1058800	1064804	26116	26299	0.025
20-	984445	990027	126847	127739	0.129
25-	888874	893914	207242	208699	0.234
30-	756616	760907	176290	177529	0.233
35-	669097	672891	117577	118404	0.176
40-	493579	496378	69540	70028	0.141
45-	408867	411186	41650	41943	0.102
50-	395244	397486	24932	25107	0.063
55-	246944	248345	10743	10819	0.044
60-65	244757	246145	4735	4768	0.019
not stated	34860	--	5663	--	--
Total	6182083	6182083	811335	811335	--

* *Source : CAPMAS :" Census of population in A.R.E. in 1986", Cairo, 1990*

Table (2)

Observed Female Educational -Specific Employment Rates For Urban Areas of A.R.E. in 1986

Educational status (1)*	Number of females (2)*	Corrected number of females (3)	Number of currently employed females (4)*	Corrected number of currently employed females (5)	$\hat{\phi}_2(x_2)$ = (5)÷(3) (6)
Illiterte	3314405	3329320	75046	75076	0.023
Read and write	876316	880259	33017	33031	0.038
Primary certificate	245638	246745	7613	7616	0.031
Less than medium certificate	596522	599207	22945	22954	0.038
medium certifciate	1111560	1116562	459850	460036	0.412
more than medium certificate	116116	116638	73413	73443	0.630
University degree and equivelants	203892	204810	136869	136925	0.669
past graduate diploma	711	714	519	519	0.727
master's degree	2602	2614	2450	2451	0.938
doctor's degree	2687	2699	2611	2612	0.968
not stated	29119	–	330	–	–
Total	6499568	6499568	814663	814663	–

* *Source: The same as in table (1)*

Table (3)

The calculations required to Derive the values of the parameters of $\phi_1(x_1;\theta_1)$ using data of Urban Areas of A.R.E. (1986).

Age Groups	x	$\frac{x-37.5}{5}$ x'	$\hat{\phi}_1(x_1)=f$	f x'	f x'^2	f x'^3	fx'^4
15-	17.5	-4	0.025	-0.100	0.400	-1.600	6.400
20-	22.5	-3	0.129	-0.387	1.161	-3.483	10.449
25-	27.5	-2	0.234	-0.468	0.936	-1.872	3.744
30-	32.5	-1	0.233	-0.233	0.233	-0.233	0.233
35-	37.5	0	0.176	0	.0	0	0
40-	42.5	1	0.141	0.141	0.141	0.141	0.141
45-	47.5	2	0.102	0.204	0.408	0.816	1.632
50-	52.5	3	0.063	0.189	0.567	1.701	5.103
55-	57.5	4	0.044	0.176	0.704	2.816	11.264
60-65	62.5	5	0.019	0.095	0.475	2.375	11.875
Total			1.166	-0.383	5.025	0.661	50.841

$\mu_1' = -0.3285$ & $\mu_2' = 4.3096$ & $\mu_3' = 0.5669$ & $\mu_4' = 43.6029$

$\mu_2 = 4.2017$ & $\mu_3 = 4.7431$ & $\mu_4 = 47.1032$

$\beta_1 = 0.3033$ & $\beta_2 = 2.6681$

$K = -0.1586$

$r = 5.2035$

$a_1 + a_2 = 10.9906$ & $a_1 = 2.1993$ & $a_2 = 8.7974$

$m_1 = 0.6393$ & $m_2 = 2.5643$ & $m_1 + m_2 = 3.2036$

$\phi_0 = 0.2185$ & Mean $= 35.8575$ & Mode $= 34.5883$

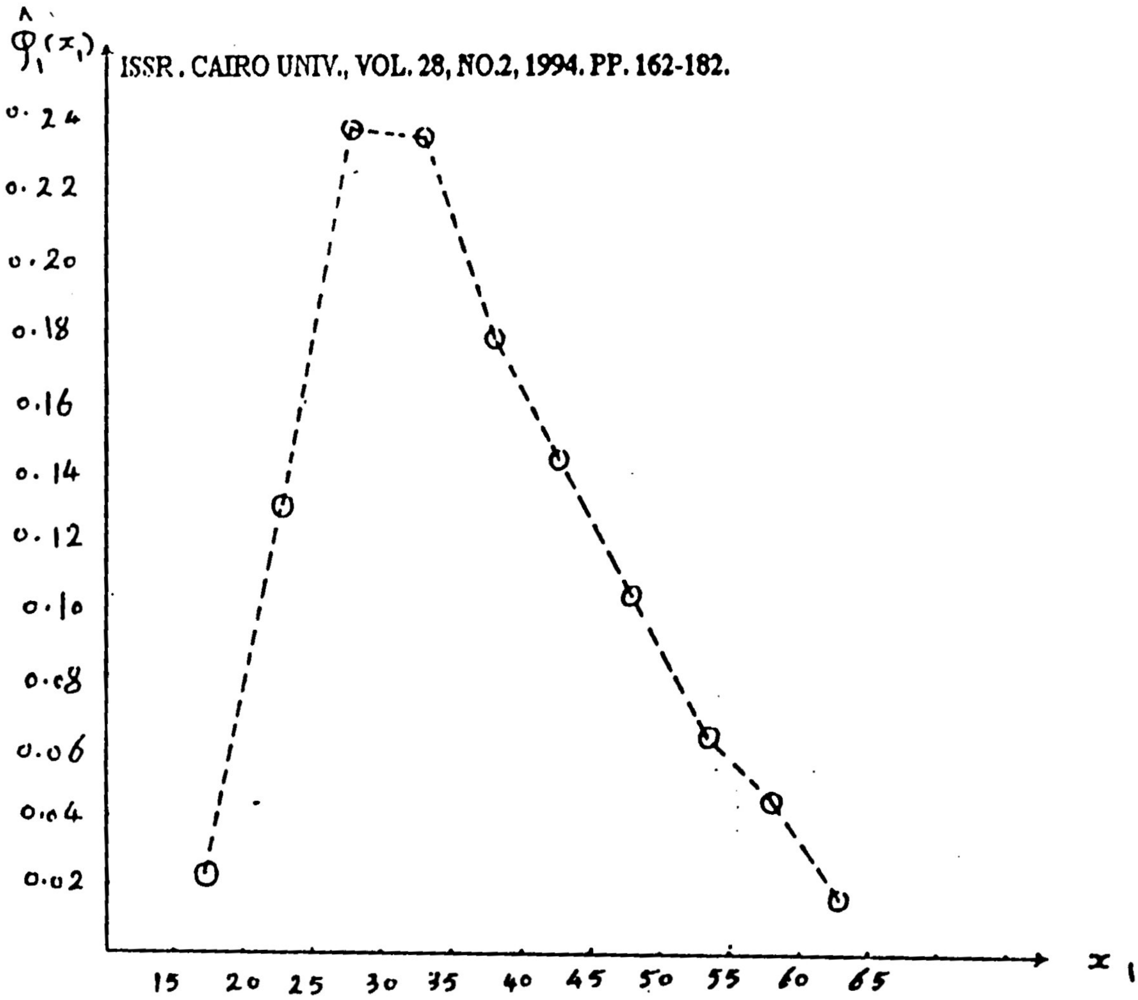

Figure (1): The observed values of the crude force of female employment as a function of age

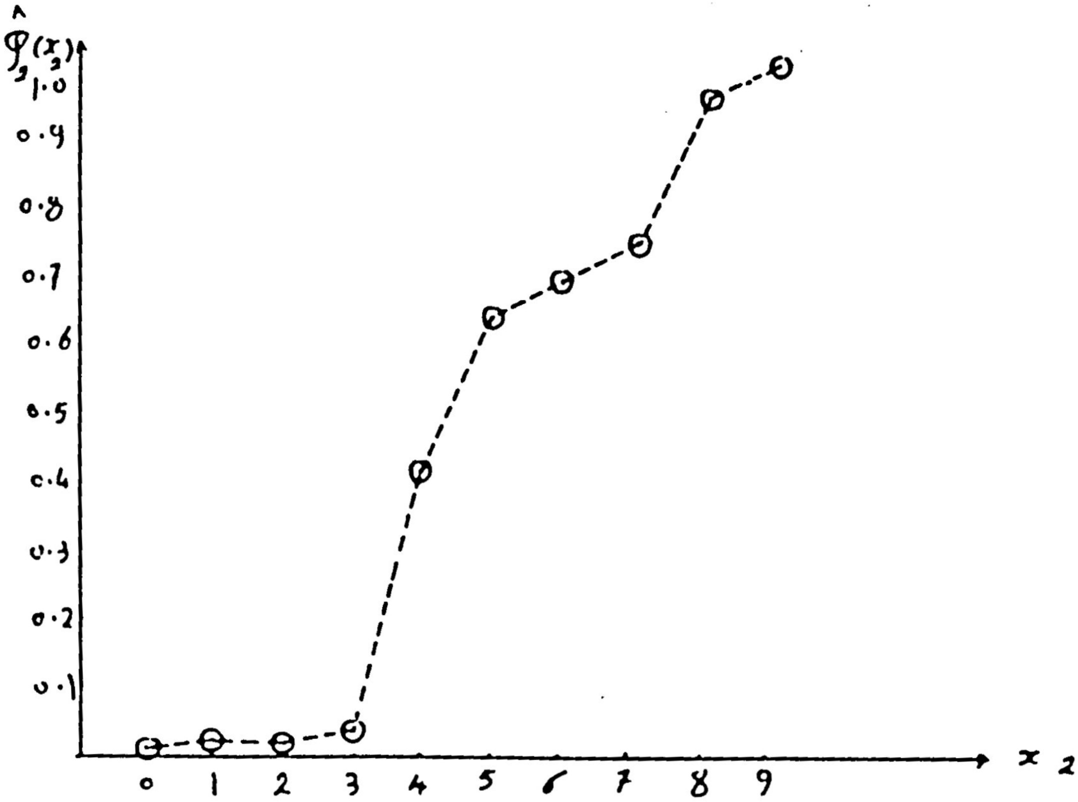

Figure (2) : The observed values of the crude force of female employment as a function of educational status.